Lecture 9 Stellar Structure

We leave the big bang now because we have so much to cover.

**Stellar Structure**

Thermonuclear reactions / hydrogen burning
Solar neutrinos
Neutrino oscillations in solar / atmospheric accelerators, neutrinos
Supernovae / supernova neutrinos
Heavy element nucleosynthesis
Neutron stars

**Stellar Structure**

Reference: Bahcall, Arnett, Chap. 6; Hauser, Chap. 2; Clayton, p.17-35 on nuclear rxns.

**Life of a Star**

Gravitational time scale is ratio of gravitational binding energy to total luminosity

t_{grav} = \frac{G M_0^2}{L_0} \leq 1.7 \times 10^7 \text{ yr.}

This is approx. equal to thermal or Kelvin-Helmholtz time scale.

Without nuclear reactions a gas cloud that starts with \( \sim 0 \) gravitational binding energy can collapse, become hot and radiate the gravitational energy away. Thus even with nuclear energy the stars can still shine, however the Sun could only shine for \( \sim 10^7 \text{ yr.} \)
after this it would start to cool off.

The nuclear time scale is the ratio of total nuclear energy

\[ t_{\text{nuclear}} \approx 0.1 \times M_\odot c^2 / L_\odot \times 10^{10} \text{ yr.} \]

Here 0.1 represents the approximate fraction of the Sun's mass that is exhausted before it leaves the main sequence and becomes a red giant.

Stars spend most of their time on the main sequence burning \( \mathbf{H} \) to \( \mathbf{He} \)

\[ 4 \mathbf{p} \rightarrow 4 \mathbf{He} + 2 \mathbf{e}^+ + 2 \nu \text{e} \]

Converts \( 4.7 \% \) of the proton's rest mass into energy. I.e., the \( \mathbf{He} \) is bound by about 25 MeV.

**Hydrostatic Equilibrium**

Assume Sun is in more or less steady state where thermal pressure balances gravity.

\[
\frac{dP(r)}{dr} = - \frac{GM(r) \rho(r)}{r^2}
\]

Enclosed mass \( M(r) = 4\pi \int r \, dr' \, r'^2 \rho(r') \)

\( \rho(r) = \text{(mass) density} \)

Temperature gradient is related to energy flux

Think of particle diffusion. If the density \( \rho \) of particles \( n(r) \) is nonuniform, there will be a current \( \mathbf{j} \)
\[ J = - \frac{4}{3} \Delta n \left( \frac{r}{\lambda} \right) \]

Here \( \lambda \) is the mean free path

\[ \lambda = \frac{1}{\sigma p} \]

For particles that can scatter from a medium of section \( \sigma \), note units \( \sigma \) with cross section \( (\sigma \sigma) = 1 \text{ L}^2 \)

So \( \sum \lambda = L \text{ L} \)

For a gas of photons, think of the energy density

\[ \epsilon = \frac{\pi^2}{15} T^4 = a T^4 \]

Instead, note in more complete units

\[ \epsilon = \frac{\pi^2}{15} \frac{k^4}{(hc)^3} T^4 \]

Thus

\[ a = \frac{\pi^2}{15} \frac{k^4}{(hc)^3} = 7.565 \times 10^{-15} \frac{\text{erg}}{\text{cm}^3 \cdot \text{K}^4} \]

Stefan–Boltzmann constant

In place of cross section \( \sigma \) one refers to the opacity \( \kappa \), which describes how photons are scattered and absorbed by the hot plasma. Thus the gradient of energy current or luminosity

\[ L \rho = -4 \pi r^2 \left( \frac{a \epsilon}{3} \right) \frac{1}{\Delta p} \frac{dT^4}{dr} \]
Here we have integrated the energy flux over a total surface area

$$S^2 = 4\pi r^2$$

to get the total luminosity [energy/time] flowing through the surface.

We need an equation of state $P = P(\rho, T)$

$$P = \frac{\alpha T^4}{3} + \frac{\mu}{M} \beta kT (1 + \delta)$$

- Radiative pressure (small)
- Ideal gas of mean molecular weight

$$M^{-1} = \frac{2x + \frac{3}{4}y + \frac{1}{2}z}{\text{Fraction H}} \frac{1}{\text{Fraction He}}$$

Hydrogen dissociates into 2 particles.

Heavy nucleus say 12C with mass 12

12C dissociates into 7 particles 6 electrons + nucleus $7/12 \neq \frac{1}{2}$

D represents easily calculated corrections from degeneracy. Sun is slightly degenerate

Invart equation of state to get density

$$\rho = \rho(P, T)$$
Finally need energy generation
\[
\frac{d\varepsilon_r}{dr} = \dot{\rho} \left( 4\pi r^2 \left[ \varepsilon_{\text{nuclear}} - T \frac{dS}{dt} \right] \right)
\]
\(\varepsilon_{\text{nuclear}} = \text{energy generation per unit mass}\)
\(\varepsilon_{\text{nuclear}} = \varepsilon_{\text{nuclear}}(\rho, T)\)

In general a very stiff function of \(T\) as we will calculate.

If no nuc. reactions Sun
\(S\) is stellar entropy. \(Tds\) describes mechanical energy generation. Also for example heavy material sinking lower. In steady state during main sequence
\[
\varepsilon_{\text{nuc}} = T ds = \dot{\varepsilon}_{\text{nuc}}.
\]

Equations of Structure
\[
\frac{dp}{dr} = -G\frac{M(r)}{r^2} \rho
\]
\[
\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon_{\text{nuc}}
\]
\[
L_r = -\frac{4\pi r^2}{3} \frac{1}{\rho} \frac{d}{dr} \left( \frac{d}{dr} \right) aT^4
\]

Also
\[
P = P(\rho, T)
\]
\[
M(r) = \frac{4\pi}{3} \int_0^r S_0 dr' r'^2 \rho(r')
\]

Start with some central \(P_0, T_0\)
\[
\begin{align*}
M(c) &= 0 \\
L_\infty &= 0 \\
P(0) &= P_c(T_c) \\
\text{Integrate out} \\
P(dr) &= \frac{dp}{d(r)} \ dr + P_c(r) \\
M(dr) &= \frac{4}{3} \pi (dr)^3 \rho_c \\
\text{This defines the surface of the star.}
\end{align*}
\]

Need appropriate boundary conditions on the luminosity and stellar atmosphere. This is a very subtle and complicated problem. However, internal structure and solar V insensitive to boundary conditions.