Lecture 13 Solar ν Production

Last time PP Chain reactions

\[ E_{\nu} \leq 0.42 \text{ MeV} \]

1. \( p + p \rightarrow d + e^+ + \nu_e \)
2. \( p + d \rightarrow \alpha + \gamma \)
3. \( ^2\text{He} + ^2\text{He} \rightarrow ^4\text{He} + 2p \)
4. \( ^3\text{He} + ^3\text{He} \rightarrow ^7\text{Be} + \gamma \)
5. \( e^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu_e \)
6. \( ^7\text{Li} + p \rightarrow 2 \alpha \)
7. \( p + ^7\text{Be} \rightarrow ^8\text{B} + \gamma \)
8. \( ^8\text{B} \rightarrow ^7\text{Be} + e^+ + \nu_e \)

\[ \stackrel{\lesssim}{L > 2.4 \times 10^{15}} \]

Many low E \( \nu_{\text{pp}} \), Some 15% \(^7\text{Be} \) \( \nu \) and very few high \( E \) \( \nu \)

\[ S \text{ (keV - barns)} \]

- \( 4.07 \times 10^{-2} \)
- \( 2.5 \times 10^{-3} \)
- \( 0.15 \times 10^3 \)
- \( 0.54 \)
- \( 5.15 \times 10^3 \)
- \( 0.861 (90\%) \)
- \( 0.0383 (10\%) \)
- \( 52 \)
- \( 0.024 \)

\[ \text{competition between reactions 1 and 4. Large S factor for 3 vs. large 4 He abundance.} \]

\[ \text{competition between electron capture 5 and proton capture 7. High} \]

\[ \text{Coulomb barrier means 7 is very rare. Flux of 8B} \nu \ll \text{flux of } ^7\text{Be} \nu \]

\[ \text{Large Coulomb barrier implies 7 is very temp. dependent.} \]
\[ f_0, \ 7 \text{Be}(p, \gamma) \]
\[ M = \frac{7 \text{Me}}{8} \]
\[ E_0 = \left( \frac{T_b}{2} \right)^{2/3} = 18.01 \text{ keV} \]
\[ b = 2 \pi \alpha Z_1 Z_2^2 \left( \frac{\text{MeV}}{\text{keV}} \right)^2 \]
\[ b = 117.6 \text{ keV} \]
\[ f_0, T = 1.3 \text{ keV} \]

Temp. dependence \( T^n \)
\[ n = \left( \frac{E_0}{T} \right)^{-\frac{2}{3}} \]

\[ n \sim 13.2 \]

at very center of Sun
and larger further out
where \( T < 1.3 \text{ keV} \).

The \( 8\beta \) flux is even more
sensitive to temp. than this because
the amount of \( 7\text{Be} \) produced
depends on temp. as follows.

The equilibrium (steady state) \( ^3\text{He} \)
abundance depends on temp. as
follows.

(a) \( ^3\text{He} \) production depends on \( R_{pp} \)
which is only weakly \( T \) dependent \([\text{He}^4] \)

(b) \( ^3\text{He} \) destruction depends on \( R_{\text{He3}}, R_{\text{He4}} \)
and \( R_{\text{He4}} \) which have
14 times the calomie barrier and are
much more strongly \( T \) dependent
For $^3\text{He} + ^3\text{He}$, $E_0 = 21.4$ keV

$n \approx 15.8$

Abundance of $^3\text{He}$

$$R_{33} n(^3\text{He})^2 + R_{34} n(^3\text{He}) n(^4\text{He})$$

$$= R_{pp} n(^4\text{H})^2$$

In steady state equilibrium, production rate = destruction rate. If $^3\text{He} + ^3\text{He} \rightarrow ^6\text{He} + ^2\text{H}$ dominates destruction

$$n(^3\text{He}) \propto \left( \frac{R_{33}}{R_{pp}} \right)^{1/2} n(^4\text{H})$$

Likewise, abundance of $^7\text{Be}$ depends on production and destruction rates

$$n(^7\text{Be}) \sim \frac{R_{34} n(^3\text{He}) n(^4\text{He})}{R_{Be} n(^5\text{Be})}$$

$$\Rightarrow n(^7\text{Be}) \propto R_{34} \left( \frac{R_{33}}{R_{pp}} \right)^{1/2} n(^4\text{H}) n(^4\text{He})$$
Finally

\[ n(\beta) \propto R_{17} n(1\text{H}) n(4\text{Be}) \]

Thus

\[ n(\beta) \propto R_{17} R_{34} \left( \frac{R_{33}}{R_{11}} \right)^{\frac{3}{2}} n(1\text{H})^2 n(4\text{He}) \]

For, \( R_{34} \)

\[ F_0 = 22.5 \text{ keV} \]

\[ n = 16.6 \]

\[ m = \left( \frac{3.4}{7} \right) m_m \]

\[ n(\beta) \propto T^{13.2} \frac{1}{T^{16.6}} \left( \frac{T^{15.8}}{T^{12}} \right) \]

\[ n(\beta) \propto T^{24} \]

Thus the flux of Böhm neutrinos is proportional to the 24th power of the temp.

A great deal of attention has been focused on pp reaction 5 factors.

1. \( R_{33} \) has been measured all the way down to 16.5 keV. This is the first time a reaction has been measured at the Gamow peak energy directly.

At this energy the cross section is 0.02 ± 0.02 pb (1 pb = 10^{-15} barns)
and the count rate was 2 events/month!

To avoid cosmic ray backgrounds and be able to see such low count rates, they needed to build a small accelerator deep underground.

See R. Bonetti et al., PRL 82 (1999) 5205. Also http://www.lngs.infn.it/site/exp.pro/luna/luna.html

LUNA = Laboratory for Underground Nuclear Astrophysics.

Much attention also on measurement of S[97] \[ p + 7Be \rightarrow 8B + \gamma \]. Original measurements used radioactive \( 7Be \) target. Now also have measurements using \( 7Be \) beam on proton target and Coulomb excitation of \( 8B \) in the Coulomb field of a Pb nucleus \[ 8B + X^+ \rightarrow 7Be + P \].

Note \( 8B \) made from \( 7Be \) so no matter what \( S \) factors or temp.

If we see some \( 8B \) neutrinos, there must be more \( 7Be \) neutrinos. We have directly seen \( 8B \) V but have not yet directly isolated \( 7Be \) V.
CNO Cycle

Sun does not yet produce much $^12_{\text{C}}$ of its own. However, gas cloud that collapsed to form Sun had been enriched in $^12_{\text{C}}$ from previous generation stars. Indeed all $^12_{\text{C}}, ^14_{\text{N}}, ^{56}_{\text{Fe}}$, etc., on Earth are from previous generation stars. Stars that died more than 4.6 BY ago (age of solar system). If some $^12_{\text{C}}$ exists than one can avoid the very small $S$ factor of the $^{12}_{\text{C}}$ reaction.

\[ a \quad p + ^{12}_{\text{C}} \rightarrow ^{13}_{\text{N}} + \gamma \]
\[ ^{13}_{\text{N}} \rightarrow ^{13}_{\text{C}} + e^+ + \nu_e \quad E_{\nu_e} \leq 1.2 \text{ MeV} \]

\[ b \quad ^{13}_{\text{C}} + p \rightarrow ^{14}_{\text{N}} + \gamma \]

\[ c \quad ^{14}_{\text{N}} + p \rightarrow ^{15}_{\text{O}} + \gamma \]
\[ ^{15}_{\text{O}} \rightarrow ^{15}_{\text{N}} + e^+ + \nu_e \quad E_{\nu_e} \leq 1.70 \text{ MeV} \]

\[ d \quad ^{15}_{\text{N}} + p \rightarrow ^{12}_{\text{C}} + \alpha \]

In CNO cycle for every $\alpha$ produced have $^1_{\text{H}} + ^{13}_{\text{N}} \nu \leq 1.2 \text{ MeV}$ and $^1_{\text{H}} + ^{15}_{\text{O}} \nu \leq 1.7 \text{ MeV}$.
CNO cycle does not produce net C, N, or O. Instead it takes 4 p into He and 2 e^+ and 2 e^- exactly as does pp cycle.

The rate limiting step is because this has largest Coulomb barrier.

\[ R_{114} \rightarrow T_{11} \]
\[ E_0 = 26.7 \text{ keV} \]
\[ n = 20 \]

\[ m = \frac{14}{15} m_\text{H} \]
\[ Z_1 = 1 \]
\[ Z_2 = 7 \]

In the Sun the CNO cycle produces only about 1.5% of the energy. However, somewhat more massive stars the temperature is higher and the CNO cycle can dominate hydrogen burning.

Finally, rare reaction

\[ \text{hep} \quad p + ^3\text{He} \rightarrow ^4\text{He} + e^+ + \nu \]

can produce highest energy, \( E_\nu \leq 18.6 \text{ MeV} \)

But flux is low because the abundance is low. I.e., hep reaction is weak.
Just like $pp \rightarrow d + e^+ \nu$
but rate for $pp$ depends on $\frac{(n(p))^2}{(n(H))^2}$
while rate for $hep$ depends on $n(H) n(He)$.
Furthermore proton has to tunnel through
a charge 2 coulomb barrier, for hep
reaction. Finally nuclear structure reduces
hep $S$ factor compared to that for $pp$
although phase space from high $E$ $\nu$
is larger.

One can also have $pp$ electron
capture

1. $pp \rightarrow d + e^+ \nu_e \quad E_\nu \leq 0.42$ MeV
1'. $p + e^+ np \rightarrow d + $ $\nu_e \quad E_\nu = 1.4$ MeV

Reaction 1' produces a higher $E$
$\nu$ by $2\text{m}e\text{c}^2$ plus the kinetic
energy of the position in 1.

\[
\frac{\text{Rate for } 1'}{\text{Rate for } 1} = \frac{\text{Phase Space for } pep}{\text{Phase Space for } pp}
\]

We will calculate this ratio next time.
<table>
<thead>
<tr>
<th>Source</th>
<th>Flux (at Earth) (10^{-6} \text{ cm}^{-2} \text{s}^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>pp</td>
<td>6.0</td>
</tr>
<tr>
<td>pep</td>
<td>0.014</td>
</tr>
<tr>
<td>hep</td>
<td>(8 \times 10^{-7})</td>
</tr>
<tr>
<td>(^{7}\text{Be})</td>
<td>0.47 (\pm) 10%</td>
</tr>
<tr>
<td>(^{8}\text{B})</td>
<td>(5.8 \times 10^{-4}) 0.01%</td>
</tr>
<tr>
<td>(^{13}\text{N})</td>
<td>0.06 (\frac{1%}{\text{of pp flux}})</td>
</tr>
<tr>
<td>(^{15}\text{O})</td>
<td>0.05 (\frac{1%}{\text{fraction of E from CN0 cycle}})</td>
</tr>
</tbody>
</table>

- \(^{7}\text{Be}\) is \(\sim 10\%\) of pp flux
- \(^{8}\text{B}\) is \(\sim 0.01\%\)
- \(^{13}\text{N}\) or \(^{15}\text{O}\) is \(\sim 1\%\)

Total pp flux is closely related (but not exactly) to solar luminosity

\[ \phi(8B) \propto T^{2.4} \text{ very temp. dependent} \]

A 2.5\% measurement of \(\phi(8B)\) constrains central temp. to \(\sim 1\%\).