Lecture 32 Neutron Stars

White dwarfs and Chandrasekhar mass.

Energy of electron gas

\[ E = 2 \int \frac{k^2}{\sqrt{k^2 + m_e^2}} \, \frac{d^3k}{(2\pi)^3} \]

\[ P = \frac{k^3}{3\pi^2} \]

In extreme relativistic limit \( k_f \gg m_e \)

\[ E \sim 2 \int k \left[ \frac{4\pi k^2 dk}{(2\pi)^3} \right] \]

\[ = \frac{3}{4} k_f P \]

Energy per particle

\[ \frac{E}{N} = \frac{3}{4} k_f \rho^{1/3} \]

Gravitational energy per particle

\[ U \sim -\frac{GM}{r} \times 5 \frac{GM}{r^{3/2}} \rho^{1/3} \]

\[ \rho = \frac{N}{V} = \frac{N}{\frac{4}{3}\pi r^3} \]
Thus as $P \to \infty$ both gravitational and kinetic energies scale as $P^{1/3}$. However, the coefficient of negative gravitational energy is proportional to $M$. If $M > M_{\text{chand}}$, the star will collapse to arbitrarily high density.

$M_{\text{chand}} \approx 1.4M_0$

**Neutron Stars**

Shortly after discovery of neutron stars in 1932, Landau proposed neutron stars. In late 1930s, J. R. Oppenheimer at Berkeley worked on structure of neutron stars.

It is an exciting time to study neutron stars. New observations are gradually turning them into detailed models.

Nevertheless, they are still very interesting theoretical curiosities with very strong gravitational and electromagnetic fields and
very dense matter.

Neutron stars are dense, known objects before matter collapses into a black hole.

Their central densities are a few 10^14 times normal nuclear density. Note, neutron stars are the only known way to study cold dense matter.

Relativistic heavy ion collisions can produce very high densities but only at high temperatures.

General relativistic corrections to neutron star star structure are large. The gravitational red shift from the surface of a neutron star is \( \approx 0.3 \).

Equations of hydrostatic equilibrium in G.R. Oppenheimer - Volkoff eq. See Misner Thorne Wheeler Gravitation for example.
In newtonian gravity

$$\frac{dp}{dr} = - \frac{GM(r) \rho(r)}{r^2}$$

$$M(r) = \int_0^r 4\pi r'^2 \, dr' \, \rho(r')$$

Enclosed mass

In G.R., $\rho$ is now the density (c = 1) and

$$\frac{dp}{dr} = - \frac{G(M(r) + 4\pi r^3 \rho)(\rho + \rho)}{r^2 \left[ 1 - 2GM(r)/r^2 \right]}$$

In nonrel. limit, $p \ll \rho$

Also, the schwartzsch. radius

$$r_s = 2GM(r)$$

Note units

$$E = \frac{GM}{r}$$

or

$$E_{\text{rest}} = \frac{GM}{r - c^2}$$

$$r_s = 2GM(r)/c^2$$
We have been using units of $c=1$

For $M = 1 \, M_\odot$ \hspace{1cm} $r_s = 1.4 \, \text{km}$

$r = 2GM/c^2 = 1.4 \, \text{km}$

Denominator correction \hspace{1cm} $1 - r_s/r$

is not important until $r \lesssim r_s$

In a neutron star $r < 10 \, \text{km}$

thus denominator correction very important.

G.R. corrections

(a) Both mass and energy and pressure contribute to gravity

(b) Gravity in G.R. is nonlinear.

This leads to the $1/r^4$ term in the denominator. These nonlinearities cause the strength of gravity to increase even faster than Newtonian gravity as object becomes more compact.

These nonlinearities lead to a maximum mass for a neutron star around $2-2.5 \, M_\odot$. 

Mass limit for white dwarf occurs because pressure for relativistic electrons only increases as $p^{4/3}$.

Mass limit for neutron stars occurs because G.R. is nonlinear and applies to any equation of state no matter how stiff.

Stiffest possible equation of state

$$p = p(p, T)$$

but for neutron stars too

thus

$$p = p(p)$$

Stiffest E.O.S. has

$$p = p$$

and the speed of sound equal to the speed of light.

Causality requires

$$p \leq p$$
The structure of a neutron star follows from the E.O.S.: only input needed.

Neutron stars are in some ways simpler than conventional stars such as the sun because we don't need $T(r)$. Therefore, no need to keep track of luminosity and opacity.

**Neutron star observables**

- Mass: Many well-measured neutron stars in binary systems have $M = 1.4 M_\odot$

- Rotational period: Neutron stars make wonderful clocks because they are almost perfect flywheels. Measure $P$ from frequency of radio pulses. In many cases also measure $\dot{P} = -\frac{dT}{dP}$ and sometimes $\ddot{P}$. We see that neutron stars are slowing down very slowly.

- Magnetic field: Star has large rotational energy $E = \frac{1}{2} I \omega^2$ and is slowing down. This energy
is thought to be radiated into EM waves which can power
the nebula and the radio pulsars. Equating change in rest energy to
power of magnetic dipole radiation yields magnetic fields of order
10^{12} gauss for many pulsars.

Rotational Glitches

As star slows down, shape of solid crust would like to change
slightly and angular momentum in crust must decrease. This leads
to sudden rotational glitches thought to occur as superfluid vortices
pinned to lattice of solid crust suddenly become unpinned.

Note neutrons in superfluid and protons may be superconductors.

X-ray spectrum

Infer surface temperature and cooling rate since NS was born
hot in a SN.
We may have finally detected two spectral lines on one neutron star. This provides information on grav. redshift,
\[ z \propto \frac{GM}{R} \]
composition of atmosphere and magnetic fields.

Neutron star radius

Is very interesting → directly provides information on EOS.
Exotic soft squishy rabbit in center such as quark matter could lead to small radius.

Measure luminosity and surface temp. → infer surface area
\[ L = 4\pi R^2 \sigma T^4 \]

For a black body. Note large curvature of space
\[ R_a = R \left(1 - \frac{2GM}{R}\right)^{-\frac{1}{2}} \]