P630 Nuclear Astrophysics

Problem Set #6 **C. Horowitz**

Due Monday Nov. 18, 2002

0] Consider neutrinos in thermal equilibrium at a temperature T and zero chemical potential. Calculate the average neutrino energy $< E_\nu >$.

a) For a Maxwell Boltzmann distribution show

$$< E_\nu >= \; 3kT$$

b) Calculate $< E_\nu >$ for a Fermi Dirac distribution.

1] Gravitational binding energy of a neutron star.

The gravitational binding energy (in Newtonian gravity) of a uniform sphere of mass M and radius R is

$$BE = \frac{3}{5}\frac{GM^2}{R}$$

a) Calculate BE for a neutron star with $M = 1.4M_\odot$ and $R = 10km$,

b) The gravitational mass of a neutron star determines the orbit of planets and binary stars. The baryon mass, M_B is the amount of material that had to collapse to form the gravitational mass. Thus

$$M_B = M + |BE|$$

Calculate M_B for this $1.4M_\odot$ star and calculate the number of baryons contained in M_B.

c) Assume the binding energy BE is radiated away in neutrinos of temperature $kT \sim 5MeV$ or from problem 0, $< E_\nu >\approx 15\,MeV$. Calculate the total # of neutrinos radiated and the number of ν radiated per baryon, part b), in the star.

2] Neutrino diffusion time in a supernova

a) Calculate the average baryon density in the neutron star. Use your baryon # from problem 1 (b) and divide by a volume $\frac{4}{3}\pi R^3$ with $R = 10km$. Show $\rho \approx 3\rho_o$ where the central baryon density of normal nuclei is $\rho_o \approx 0.15\,Fm^{-3}$.

b) The neutrino mean free path is

$$\lambda = \frac{1}{\sigma\rho}$$

Assume
$$\sigma = \frac{G_F^2}{\pi}(g_\nu^2 + 3g_a^2)E_\nu^2$$

with G_F, the Fermi constant, $g_\nu = 1$ and $g_a = 1.26$. Let the temperature inside the supernova be $kT \approx 10\ MeV$ so $< E_\nu > \approx 30\ MeV$. Calculate λ assuming the average density from part a). Note the temperature inside is greater than the surface temperature.

c) The neutrinos will escape from the star in a time of order

$$\tau \sim \frac{R^2}{\lambda c}$$

This is the time to diffuse a distance $R = 10km$. Calculate τ using λ from part (b). Note, the neutrino signal from a supernova will have a duration roughly characterized by τ. For example, the signal could be a decaying exponential with a decay constant of order τ. Compare with the $\sim$ 10 seconds over which ν were detected from SN 1987a.

3] Election capture in dense matter.

a) Consider pure hydrogen at low temperature. Calculate the threshold density for electron capture,

$$e^- + p \rightarrow n + \nu_e$$

b) Assume the hydrogen contains a small amount of ^3He. Calculate the threshold density for,

$$e^- + {}^3He \rightarrow {}^3H + \nu_e$$

Note the mass of ${}^3H$ is 18 keV above the mass of ${}^3He$ and an electron.

4] The URCA process in neutron stars

Consider the reaction,

$$n \rightarrow p + e^- + \bar{\nu}_e,$$

followed by,

$$e^- + p \rightarrow n + \nu_e,$$

in the interior of a neutron star at low temperatures. Assume the energy of the emitted ν_e or $\bar{\nu}_e$ is very small compared to the neutron Fermi energy. Also assume a n very near its Fermi surface beta decays to a proton and an electron both very near their respective Fermi surfaces. Show that momentum conservation requires the electron fraction (electrons per baryon, Y_e) to be greater than 1/9. Note charge neutrality says the proton and electron Fermi momenta are equal so you can show

$$Y_e = k_F^{p3} / (k_F^{p3} + k_F^{n3})$$

Here k_F^n, k_F^p are the neutron and proton Fermi momenta.