

P630 Nuclear Astrophysics

Problem Set #4

C. Horowitz

Due Monday Oct. 21, 2002

1] The pep reaction in the Sun.

a) Low energy pp solar neutrinos have a maximum energy $E_\nu \leq 0.42 \text{ MeV}$. The phase space for the final states in the $pp \rightarrow de^+\nu$ reaction can be written

$$F_{pp} = \int \frac{d^3 p_e}{(2\pi)^3} \frac{d^3 p_\nu}{(2\pi)^3} (2\pi)^4 \delta(Q - E_e - E_\nu)$$

Here $Q = 0.42 \text{ MeV} + m_e c^2$

$$Q = 0.93 \text{ MeV}$$

Evaluate F_{pp} including looking up the necessary integrals.

Note, the original $C\ell$ experiment has a 0.8 MeV threshold and therefore is insensitive to the pp neutrinos. Instead, the reaction $p + e + p \rightarrow d + \nu_e$ produces a higher ν_e .

$$Q_{pep} = Q + m_e c^2 = 1.4 \text{ MeV}$$

This neutrino can be seen by the $C\ell$ experiment,

b) The phase space for the pep reaction can be written,

$$F_{pep} = \int \frac{d^3 p_e}{(2\pi)^3} n(p_e) \int \frac{d^3 p_\nu}{(2\pi)^3} (2\pi)^4 \delta(Q_{pep} - E_\nu).$$

In the energy conserving delta function we have neglected the thermal energy of the electron (of order $kT \approx 1.3 \text{ KeV}$) compared to Q_{pep} . In F_{pep} , $n(p_e)$ is the Fermi-Dirac distribution of electrons in the Sun. The total electron number density is,

$$\rho_e = 2 \int \frac{d^3 p_e}{(2\pi)^3} n(p_e)$$

where the two come from the spin sum. Evaluate F_{pep} .

c) Coulomb corrections: The ratio of the pep ν flux to the pp ν flux is

$$R_{pep/pp} = C F_{pep}/F_{pp}.$$

Here C is a coulomb correction. Coulomb interactions increase the electron density at the nucleus by a factor

$$C = \left[\frac{2\pi\eta}{e^{2\pi\eta} - 1} \right]$$

with

$$\eta = Z_1 Z_2 \alpha / v$$

For the electron density in pep , $Z_1 = -1$ (electron) and we can take $Z_2 \approx 2$ two protons nearby in a virtual 2He state. For large negative η

$$C \approx 2\pi|\eta| = 4\pi\alpha \frac{1}{v}$$

Average $\frac{1}{v}$ over a Maxwell-Boltzmann distribution of electrons in the solar plasma at temperature $kT_c \approx 1.3 \text{ keV}$, and evaluate C as a function of temp.

d) Evaluate $R_{pep/pp}$ for $kT_c = 1.3 \text{ keV}$ and a density of 150 g/cm^3 (assume pure H). [See Bahcall Eq.(3.17)]

2] Neutrino Oscillations

Let an electron flavor neutrino be a mixture of mass m_1 and mass m_2 mass eigenstates. Assume the neutrino has a momentum k .

$$|\nu_e \rangle = \cos \theta |\nu_1 \rangle + \sin \theta |\nu_2 \rangle$$

Calculate the probability p_ν the $|\nu_e\rangle$ remains an electron flavor $|\nu_e\rangle$ after it has propagated for a time t or distance $L = ct$.

a) Show

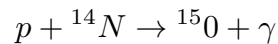
$$p_\nu(L) = 1 - \sin^2 2\theta \sin^2 \left[\frac{\delta m^2 L}{4k} \right]$$

Here $\delta m^2 = m_2^2 - m_1^2$.

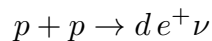
b) If the first minimum in p_ν occurs at the Earth Sun distance $L = 1 \text{ a.u.}$ (astronomical unit) for $k = 1 \text{ MeV}$, calculate δm^2 .

3] CNO versus pp cycles.

In the Sun the CNO cycle only provides about 1% of the luminosity. Calculate the temperature at which the rate for



becomes comparable to that for



assume

- (1) The mass fraction of ${}^{14}\text{N} \approx 0.01$
- (2) $S_{pp} = 4.1 \times 10^{-22} \text{ keV} - \text{ barns}$
- (3) $S_{pN} = 3.3 \text{ keV} - \text{ barns}$
- (4) The mass fraction of ${}^4\text{He}$ is 0.3 and the density is 150 g/cm^3

You can make a small table of rates versus temperature to approximately find the cross over temperature. You can neglect the energy dependence of the S factors.