P630 Nuclear Astrophysics

Problem Set #4 **C. Horowitz**

Due Monday Oct. 21, 2002

1] The pep reaction in the Sun.

a) Low energy pp solar neutrinos have a maximum energy $E_v \leq 042 MeV$ The phase space for the final states in the $pp \to de^+\nu$ reaction can be written

$$F_{pp} = \int \frac{d^3p_e}{(2\pi)^3} \frac{d^3p_v}{(2\pi)^3} (2\pi)^4 \, \delta(Q - E_e - E_v)$$

Here Q=0.42 MeV + $m_e c^2$

Q=0.93 MeV

Evaluate F_{pp} including looking up the necessary integrals.

Note, the original $C\ell$ experiment has a 0.8 MeV threshold and therefore is insensitive to the pp neutrinos. Instead, the reaction $p + e + p \to d + \nu_e$ produces a higher ν_e.

$$Q_{pep} = Q + m_e c^2 = 1.4 \, MeV$$

This neutrino can be seen by the $C\ell$ experiment,

b) The phase space for the pep reaction can be written,

$$F_{pep} = \int \frac{d^3p_e}{(2\pi)^3} \, n(p_e) \int \frac{d^3p_v}{(2\pi)^3} (2\pi)^4 \delta(Q_{pep} - E_v).$$

In the energy conserving delta function we have neglected the thermal energy of the electron (of order $kT \approx 1.3 \, KeV$) compared to Q_{pep}. In F_{pep}, $n(p_e)$ is the Fermi-Dirac distribution of electrons in the Sun. The total electron number density is,

$$\rho_e = 2 \int \frac{d^3p_e}{(2\pi)^3} \, n(p_e)$$

where the two come from the spin sum. Evaluate F_{pep}.

c) Coulomb corrections: The ratio of the pep ν flux to the pp ν flux is

$$R\,pep/pp = C\ F_{pep}/F_{pp}.$$

Here C is a coulomb correction. Coulomb interactions increase the electron density at the nucleus by a factor

$$C = \left[\frac{2\pi\eta}{e^{2\pi\eta} - 1}\right]$$

with

$$\eta = \mathrm{Z}_1\mathrm{Z}_2\alpha\,/\,v$$

For the electron density in pep, $Z_1 = -1$ (electron) and we can take $\mathrm{Z}_2 \approx 2$ two protons nearby in a virtual ^2He state. For large negative η

$$C \approx 2\pi|\eta| = 4\pi\alpha\frac{1}{v}$$

Average $\frac{1}{v}$ over a Maxwell-Boltzmann distribution of electrons in the solar plasma at temperature $kT_c \approx 1.3\ keV$, and evaluate C as a function of temp.

d) Evaluate $R_{pep/pp}$ for $kT_c = 1.3\ keV$ and a density of 150 g/cm^3 (assume pure H). [See Bahcall Eq.(3.17)]

2] Neutrino Oscillations

Let an electron flavor neutrino be a mixture of mass m_1 and mass m_2 mass eigenstates. Assume the neutrino has a momentum k.

$$|\nu_e >= \cos\,\theta\,|\nu_1 > +\sin\,\theta\,|\nu_2 >$$

Calculate the probability p_ν the $|\nu_e\ >$ remains an electron flavor $|\nu_e >$ after it has propagated for a time t or distance $L\ =\ t$.

a) Show

$$p_v(L) = \ 1 - \sin^2 2\theta\ \sin^2\left[\frac{\delta m^2 L}{4k}\right]$$

Here $\delta m^2 = m_2^2 - m_1^2$.

b) If the first minimum in p_v occurs at the Earth Sun distance $L\ =\ 1a.u.$ (astronomical unit) for $k\ =\ 1MeV$, calculate δm^2.

3] CNO versus pp cycles.

In the Sun the CNO cycle only provides about 1% of the luminosity. Calculate the temperature at which the rate for

$$p + {}^{14}N \rightarrow {}^{15}0 + \gamma$$

becomes comparable to that for

$$p + p \rightarrow d\, e^+ \nu$$

assume

(1) The mass fraction of ${}^{14}N\ \approx\ 0.01$

(2) $S_{pp} = 4.1 \times 10^{-22}\ \ keV - barns$

(3) $S_{pN} = 3.3\ keV - barns$

(4) The mass fraction of ${}^4He$ is 0.3 and the density is 150 g/cm^3

You can make a small table of rates versus temperature to approximately find the cross over temperature. You can neglect the energy dependence of the S factors.