P630 Nuclear Astrophysics

Problem Set #3 **C. Horowitz**

Due Monday Oct. 7, 2002

1] Critical density of the Universe.

The critical energy density for a flat universe is

$$\rho_{crit} = \frac{3 \quad H^2}{8\pi G}$$

Show

$$\rho_{crit} \approx \; 9.7 \times 10^{-27} \;\; kg/m^3$$

$$= \; 5400 \; eV/cm^3$$

assuming a Hubble constant H of

$$H = 72 \; km/s \;\; /MPC$$

Here a mega parsec is

$$1 \;\; MPC = 3.086 \times 10^{24} \, cm$$

2] Photon number density of the Universe.

a) If the present day cosmic microwave background has a temperature of 2.7K show the number density of photons is

$$\rho_\gamma \approx 400 \; cm^{-3}$$

b) If the neutrino background has a temperature $T_\nu \approx (4/11)^{1/3}\, T_\gamma$ show the neutrino number density per flavor is

$$\rho_\nu \approx 110\ cm^{-3}$$

c) If the baryon to photon ratio is $\eta \approx 6 \times 10^{-10}$ (from the measured deuterium abundance) use part a) and your results from problem 1 to calculate the fraction of the critical density Ω_b in baryons.

d) If the sum of all three neutrino masses is

$$M_{tot} = \sum_{i=i}^{3} m_i = 6eV$$

calculate the fraction of the critical density in neutrinos Ω_ν.

e) Assume nonbaryonic dark matter makes up a fraction 0.26 of the critical density. If the dark matter is made up of 1 TeV mass weakly interacting massive particles (WIMPs) calculate their number density.

3] Thermal Average of Reaction Rates

$$< \sigma v >= \frac{\int d^3 k e^{-E/T} \sigma(E) v}{\int d^3 k e^{-E/T}}$$

a) If $\sigma = S_o e^{-2\pi\eta}/E \quad show$

$$< \sigma v >= \left(\frac{8}{\pi \mu T^3} \right)^{1/2} \int_o^\infty dE\ S_o e^{-(2\pi\eta + E/T)}$$

with μ the reduced mass of the two ions and $\eta = \mathrm{Z_1 Z_2}\, e^2/(\hbar v)$

b) Define $I = \int_o^\infty dE\ e^{-F(E)}$ with

$$F(E) = 2\pi\eta + E/T$$

expand $F(E)$ about its minimum valve,

$$F(E) \approx F(E_o) + \frac{1}{2} F'' |_{E_o} (E - E_o)^2.$$

Here E_o is the energy where

$$dF/dE\,|_{E_o} = 0. \qquad Find \quad E_o.$$

Evaluate the resulting Gaussian integral for I by extending the lower limit to $-\infty$ and obtain an approximate value for $< \sigma v >$. Show

$$< \sigma v > \approx \left(\frac{2}{\mu T^3}\right)^{1/2} S_o \Delta\, e^{-3E_o/T} \qquad Where \quad \Delta = \frac{4}{\sqrt{3}}(E_o T)^{1/2}.$$

4] Lifetime of the Sun

The lifetime of a proton against $p + p \rightarrow d + e^+ + \nu_e$ is

$$T = \frac{1}{\rho_p < \sigma v >_{pp}}$$

Here ρ_p is the number density of protons. Assume $S_o \approx 4.1 \times 10^{-22}\, keV$ barns. For the pp reaction and the central density of the Sun is 150 g/cm^3 at a temperature of $1.6 \times 10^7 K$. Calculate T, for simplicity assume the material is pure hydrogen.