P630 Nuclear Astrophysics

Problem Set #2 **C. Horowitz**

Due Monday Sept. 23, 2002

1] Let the potential between two nucleons be,

$V(r) = V_0 + \vec{\sigma}_1 \cdot \vec{\sigma}_2 \, V_1 \qquad r < r_o$

or $V(r) = 0 \qquad\qquad\qquad r > r_o$

Here $\vec{\sigma}_{1,} \vec{\sigma}_2$ are Pauli spin matrices for nucleon 1 and 2. Let $r_o = 1Fm$. Find the constants V_o and V_1 so that two neutrons have a bound state at zero energy while a proton and a neutron are bound by 2.2 MeV into the deuteron.

Note this potential is isospin invariant. Nevertheless, it binds two nucleons in an isospin zero state into the deuteron but just fails to bind two nucleons in an isopsin one state (the dineutron or ^2He). Discuss.

In reality the strong spin dependence of the one pion exchange interaction between nucleons is responsible for deuterium being bound but ^2He being unbound.

Again, if ^2He were bound the Sun would burn hydrogen many orders of magnitude faster.

2] Consider the semi-empirical mass formula with

$E = -a_1 \, A + a_2 A^{2/3} + a_3 \mathrm{Z}^2 A^{-1/3}$

$\quad + a_4 \, (A - 2\mathrm{Z})^2 / A$

and $\quad a_1 = 15.75 \, MeV$

$a_2 = 17.8 \, MeV$

$a_3 = 0.710 \, MeV$

$a_4 = 23.7 \, MeV$

(a) For a given A minimize the energy with respect to Z and show the most stable charge is about,

$$Z^* = A\left[2 + \frac{a_3 A^{2/3}}{2\,a_4}\right]^{-1}$$

(b) Plot the energy per particle E/A versus A for $10 < A < 250$ assuming $Z = Z^*(A)$. Find the most stable mass number A^* and compare with $A^* = 56$ for $56Fe$.

3] Consider the pairing term in the semi-empirical mass formula

$$E_5 = \lambda\, a_5\, A^{-3/4}$$

with $a_5 = 34\,MeV$ and $\lambda = 1$ for odd N, odd Z, O for odd-even and -1 for even N, even Z. Explain why pairing tends to make even Z elements even more abundant than odd Z elements.

4] Calculate the total number of light degrees of freedom in the Universe.

At say $t = 0.01$ sec. after the Big Bang the temperature was

$$T \sim 10\ MeV$$

this is much less than the mass of the muon. Therefore, there will be large numbers of nearly massless photons, electrons, ν_e, ν_μ, ν_τ and (where appropriate) their antiparticles.

Assume the chemical potential for e^- and all flavors of neutrinos are small.

Calculate the total energy density E_{tot} and show

$$E_{tot} = N_{eff}\,\frac{\pi^2}{30}\,T^4$$

(A) Explain why N_{eff} is the number of boson degrees of freedom plus 7/8 of the number of fermion degrees of freedom.

(B) N_{eff} includes appropriate spin degeneracies. Explain why the spin degeneracy is two for photons or electrons, but only one for neutrinos.

(C) Show $N_{eff} = 43/4$

Note integrals over zero chemical potential Fermi-Dirac and Bose-Einstein distributions can be written in terms of gamma and zeta functions. I include an appendix from Fetter & Walecka.