9/29/00

Lecture 15 Small amplitude motion

$$q_\sigma = q_\sigma^0 + \eta_\sigma$$

static solution

$$V \simeq V_0 + \frac{1}{2}\sum_{\sigma\lambda} V_{\sigma\lambda}\eta_\sigma\eta_\lambda \qquad V_{\sigma\lambda} = \left.\frac{\partial^2 V}{\partial q_\sigma \partial q_\lambda}\right|_{q^0}$$

$$T = \frac{1}{2}\sum_{\sigma\lambda} m_{\sigma\lambda}\dot\eta_\sigma\dot\eta_\lambda$$

$$m_{\sigma\lambda} = \sum_i m_i \left.\frac{\partial x_i}{\partial q_\sigma}\right|_{q^0}\left(\frac{\partial x_i}{\partial q_\lambda}\right)_{q^0} = m_{\lambda\sigma}$$

Lagrange's equations $\sum_\lambda [\eta_\lambda V_{\sigma\lambda} + m_{\sigma\lambda}\ddot\eta_\lambda] = 0$

$$\sum_\lambda [V_{\sigma\lambda} - \omega^2 m_{\sigma\lambda}]\eta_\lambda^0 = 0$$

$$\eta_\lambda(t) = \mathrm{Re}\,\eta_\lambda^0 e^{i\omega t}$$

Normal mode frequency ω

Normal mode frequencies ω^2 satisfy

$$\det|V_{\sigma\lambda} - \omega^2 m_{\sigma\lambda}| = 0$$

N^{th} order polynomial has n real roots

$$\omega_s^2 \qquad s = 1, \ldots, n.$$

Stable if all $\omega_s^2 > 0$

Unstable if any $\omega_s^2 < 0$

General solution is a superposition of normal modes with amplitude $C^{(s)}$ and phase ϕ_s

$$\mathrm{Re}\ \eta_\lambda^0 e^{i\omega_s t} = \rho_\lambda^s \cos(\omega_s t + \phi_s)$$

Note $\rho_\lambda^s e^{i\phi_s}$ is a solution to

$$\sum_\lambda \left[V_{\sigma\lambda} - \omega_s^2 m_{\sigma\lambda}\right] \rho_\lambda^s e^{i\phi_s} = 0$$

Thus ρ_λ^s is an eigenvector with eigenvalue ω_s^2

$$\boxed{\eta_\lambda(t) = \sum_{s=1}^{n} C^{(s)} \rho_\lambda^{(s)} \cos(\omega_s t + \phi_s)}$$

Important equation. Any small amplitude motion can be written as above. Chose $C^{(s)}$ and ϕ_s $s=1,\ldots,n$ to satisfy inital conditions $\eta_\sigma(t=0)$ and $\dot{\eta}_\sigma(t=0)$ $\sigma=1,\ldots n$.

<u>Example</u> Small amplitude osc. of Sun see SOHOWWW.NASCOM.Nasa.Gov

$\rho_\lambda^{(s)}$ are fixed eigenvectors which are independent of inital conditions

<u>Review</u> Linear algebra, Matrix inverse det, eigenvectors and solutions of linear equations.

Start problem 3.19

Work problem 4.1 See solutions PSet 5 1999