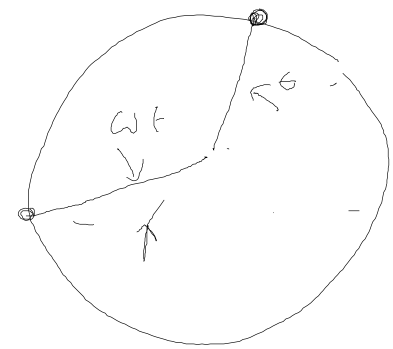
9/25/00

Lecture 13

Example of H — bead on rotating wire

$$L = T = \tfrac{1}{2} m a^2 \left[\omega^2 + (\omega + \dot\theta)^2 + 2\omega (\omega + \dot\theta)\cos\theta \right]$$



$$p_\theta = \frac{\partial L}{\partial \dot\theta} = m a^2 (\omega + \dot\theta) + m a^2 \omega \cos\theta$$

$$H = p_\theta \dot\theta - L$$

$$H = m a^2 \left[\tfrac{1}{2} \dot\theta^2 - \omega^2 (1 + \cos\theta) \right] = \text{constant}$$

$$E = T \neq \text{constant}.$$

Work problem 3.9 from problem set 3 about small osc. around stable orbit

Start reading Chapter 4 on Small Oscillations

General problem to consider stability analysis. Do amplitude of osc. of a bridge grow with time? Is an orbit of a planet stable against small perturbations? Indeed is solar system stable? Answer is no!

We will generalize to a system of many degrees of freedom.

Also get characteristic frequencies of system. Heleoseismology studies in detail spectrum of collective osc. of Sun

Examples in Nuclear physics: Is a nucleus stable against small deformations in shape?

Answer closed shell nuclei such as ^{208}Pb with $N=126$ neutrons and $Z=82$ protons are. Many open shell nuclei such as ^{238}U are unstable. Indeed true ground state is not spherical but deformed

Something like 30% large on one axis. Note ^{238}U fissions along short axis

What are some collective osc. of nuclei
$\Rightarrow$ Giant dipole resonance has neutrons osc. against protons. Shows up as a large peak in photoabsorption cross section at $E \sim 10-20$ MeV

If you make a spherical model of ^{238}U discover that some mode frequencies are complex

$e^{i\omega t}$ for complex ω can grow with time.

If you have a stable ground state then frequencies correspond to collective modes and ω^2 are all positive

Small Oscillations

Formulation ① Look for static solutions of Lagrange's equations.

② Expand coordinates around static solution to lowest order in small deviations

③ Look for osc. solutions $e^{i\omega t}$ and solve for normal mode frequencies ω. If all $\omega^2 > 0$ stable. If any $\omega^2 < 0$ than unstable.

Lagrange's equations can be written

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_\sigma} - \frac{\partial T}{\partial q_\sigma} = Q_\sigma = -\frac{\partial V}{\partial q_\sigma}$$

Static solution $\qquad \dot{q}_\sigma = \ddot{q}_\sigma = 0$

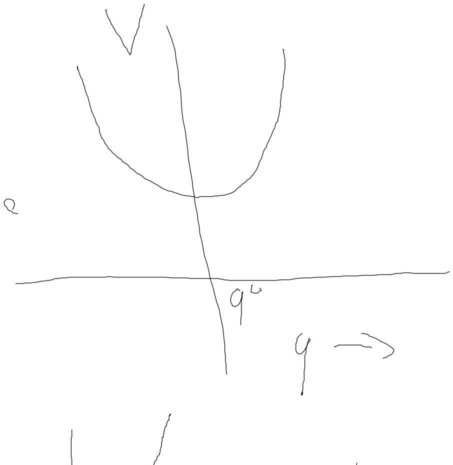
$$q_\sigma = q_\sigma^0$$

Generalized force vanishes

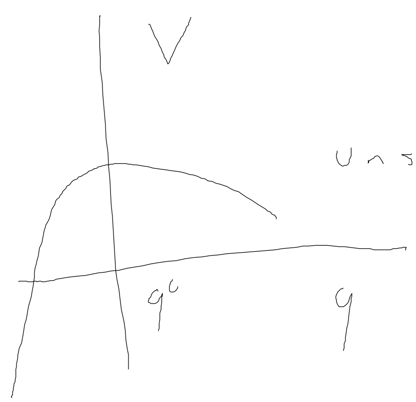
$$Q_\sigma = -\left(\frac{\partial V}{\partial q_\sigma}\right)_{q_\sigma^0} = 0$$

If force did not vanish it would push particles so $\dot{q}_\sigma \neq 0$

Stable



unstable



unstable