9/8/00

Lecture 6

Comet velocity relative to Sun

$$V_c = \sqrt{\frac{2 G M_\odot}{R_{SE}}}$$

Earth orbital velocity (circular orbit)

$$E = +\frac{1}{2} V = V + T \qquad E = -\frac{m\gamma}{2a}$$

$$V = -\frac{m\gamma}{a} = -\frac{GMm}{r}$$

$$T = -\frac{V}{2} = \frac{G M_\odot M_E}{2 R_{SE}} = \frac{1}{2} M_E V_E^2$$

$$V_E^2 = \frac{G M_\odot}{R_{SE}} \qquad V_E = \frac{1}{\sqrt{2}} V_c$$

Assume comet has very small impact parameter (not quantitative)

$$V_{Rel} = \sqrt{V_c^2 + V_E^2}$$

$$V_{Rel} = \left(1 + \frac{1}{2}\right)^{1/2} V_c$$

$$V_{rel} = \sqrt{\frac{3}{2}}\, V_c$$

$$= \sqrt{\frac{3 G M_\odot}{R_{SE}}}$$

Put in some #s

$M_\odot = 1.989 \times 10^{33}$ g

$R_{se} = 1\,\mathrm{AU} = 1.496 \times 10^{13}$ cm

$M_e = 5.976 \times 10^{27}$ g

$R_e \simeq 6.36 \times 10^{8}$ cm

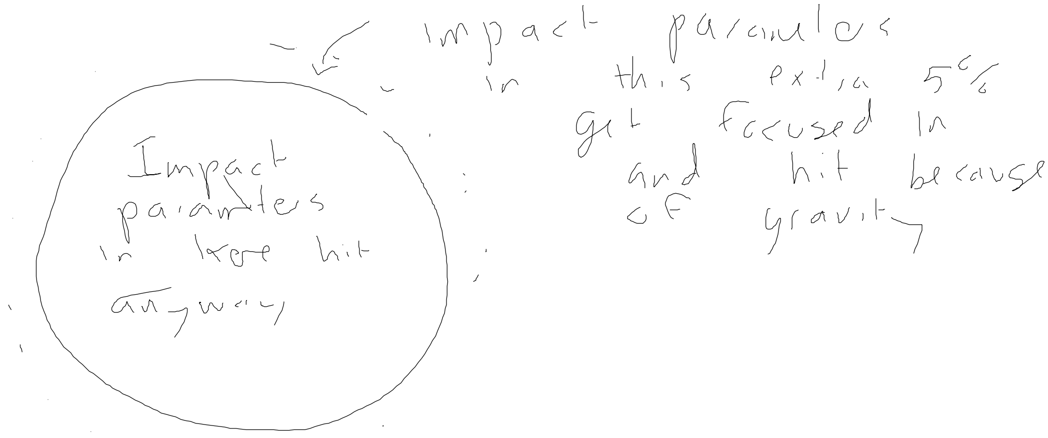
$G = 6.67 \times 10^{-8}$ dyn cm^2/g^2

$$V_{rel} = \left(\frac{3 \quad 6.67 \times 10^{-8} \quad 1.989 \times 10^{33}}{1.496 \times 10^{13}} \right)^{1/2} = 5.16 \times 10^{6} \text{ cm/s}$$

$$\sigma = \pi R_E^2 \left[1 + \frac{2 G M_e}{V_\infty^2 R_e} \right]$$

$$= \pi R_E^2 \; 1.047$$

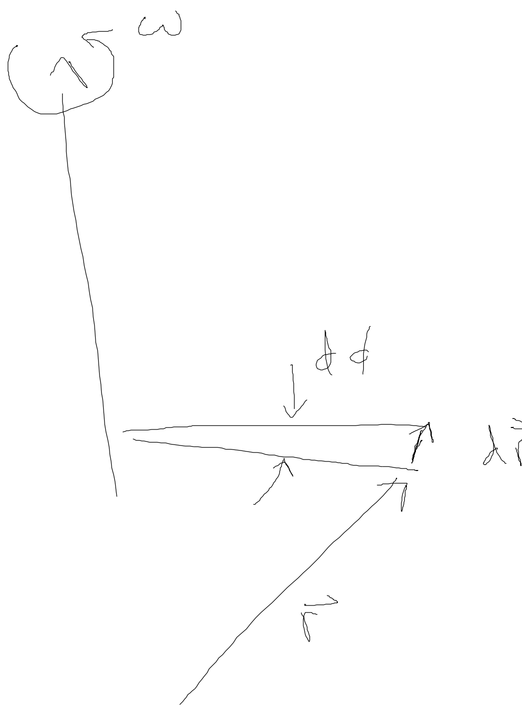
The cross section is 5% greater because of the Earth's gravity



Only a 5% correction because comets are moving fast.

Start Reading Chap. 2 Accelerated Coordinate Systems

Consider a rotating coordinate system (Earth) chose Z axis along rotation axis



$$d\vec{\Omega} = d\phi\,\hat{z}$$ small rotation about Z axis

$$\Delta\vec{r} = d\vec{\Omega} \wedge \vec{r}$$

Change in $\vec{r}$ is $\perp$ to both $\vec{r}$ and $d\vec{\Omega}$

In Earth or body fixed frame have vector $\vec{r}$

In inertial frame have vector

$$\vec{r}_{inertial} = \vec{r}_{body} + \Delta\vec{r}$$

$$\frac{d\vec{r}_{inertial}}{dt} = \frac{d\vec{r}_{body}}{dt} + \frac{d\,\Delta\vec{r}}{dt}$$

$$= \frac{d\vec{r}_{body}}{dt} + \frac{d\vec{\Omega}}{dt} \wedge \vec{r}_{body}$$

and for any vector V

$$\boxed{\frac{d}{dt}\vec{V}_{inertial} = \frac{d(\vec{V})}{dt}_{body} + \frac{d\vec{\Omega}}{dt} \wedge \vec{V}}$$

$\frac{d\Omega}{dt} = \omega$ = rotation rate

symbolic relation

$$\left(\frac{d}{dt}\right)_{inertial} = \left(\frac{d}{dt}\right)_{body} + \vec{\omega} \wedge$$

apply it twice

$$\frac{d^2(\vec{r})}{dt^2}_{inertial} = \left[\frac{d}{dt} + \vec{\omega} \wedge\right]\left[\left(\frac{d\vec{r}}{dt}\right)_{body} + \vec{\omega} \wedge \vec{r}\right]$$

$$= \left(\frac{d^2\vec{r}}{dt^2}\right)_{body} + 2\,\omega \wedge \left(\frac{d\vec{r}}{dt}\right)_{body} + \frac{d(\vec{\omega})}{dt} \wedge \vec{r}$$

$$+ \vec{\omega} \wedge (\vec{\omega} \wedge \vec{r})$$

$$m\left(\frac{d^2 r}{dt^2}\right)_{inertial} = \vec{F} = m\left(\frac{d^2\vec{r}}{dt^2}\right)_{body} + 2m\vec{\omega} \wedge \left(\frac{d\vec{r}}{dt}\right)_{body}$$

$$+ m\vec{\omega} \wedge (\vec{\omega} \wedge \vec{r})$$

or

$$m\left(\frac{d^2\vec{r}}{dt^2}\right)_{body} = \vec{F} - 2m\vec{\omega}\wedge\left(\frac{d\vec{r}}{dt}\right)_{body} - m\vec{\omega}\wedge(\vec{\omega}\wedge\vec{r})$$

In a noninertial frame Newton's Laws get modified. One can simulate the changes in the coordinate system by adding two fake forces