10/25/00

Lecture 26 Review

Midterm covers chapters 1-5
Short problems and some definitions

Chapter 1 Basic principles

Conserved quantities E, L

Write $T = T' + T_{cm}$

and $L = L' + L_{cm}$

where $L_{cm} = \vec{R} \wedge M \dot{R}$ etc.

Scattering cross section

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\Theta} \left| \frac{db}{d\Theta} \right|$$

b = impact parameter, Θ = scattering angle

Chapter 2 Accelerated coordinate systems

$$\left(\frac{d\vec{V}}{dt}\right)_{\text{Inertial}} = \left(\frac{d\vec{V}}{dt}\right)_{\text{body}} + \vec{\omega} \wedge \vec{V}$$

Newton's laws in accelerated frame

$$m\left(\frac{d^2 r}{dt^2}\right)_{\text{body}} = F^{(e)} - 2m\,\underbrace{\omega \wedge \left(\frac{d\vec{r}}{dt}\right)_{\text{body}}}_{\text{Coriolis force}} - m\,\underbrace{\omega \wedge (\omega \wedge \vec{r})}_{\text{centrifugal force}} - m\left(\frac{d\omega}{dt}\right) \wedge \vec{r} - m\left(\frac{d^2 a}{dt^2}\right)_{\text{Inertial}}$$

Chapter 3 Lagrangian Dynamics

$$L = T - V \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\sigma}\right) - \frac{\partial L}{\partial q_\sigma} = 0$$

$$\sigma = 1, \dots, N$$

(i) Incorporate constraints and avoid dealing explicitly with forces of constraint
(ii) Rewrite dynamics in any convient generalized coordinates

Review derivation of Lagrange's eq.
(a) From D'alembert's principle → Forces of constraint do no work under a virtual displacement
(b) Minimize action using calculus of variations.

Definitions

- Holonomic constraints $f(x_1 \dots x_n, t) = c$
- Generalized coordinates
- Virtual displacements
- D'alembert's principle
- Hamilton's principle
- Forces of constraint
- Lagrange multipliers
- ...

Symmetries and conserved quantities

Generalized momenta $p_\sigma \equiv \frac{\partial L}{\partial \dot{q}_\sigma}$

If $\frac{\partial L}{\partial q_\sigma} = 0$ than q_σ is a cyclic coordinate

and $\frac{d}{dt} p_\sigma = 0$

Hamiltonian $H \equiv \sum_\sigma p_\sigma \dot{q}_\sigma - L$

If Lagrangian does not depend explicitly on time $\frac{\partial L}{\partial t} = 0 \Rightarrow \frac{dH}{dt} = 0$

If pot. is time independent and constraints are time-independent $\Rightarrow$ $H = T + V = E$

Chapter 4 Small Osc.

Very general!

Static equilibrium $\left.-\left(\frac{\partial V}{\partial q_\sigma}\right)\right|_{q^0} = 0$ $\sigma = 1, \dots, n$

so $\dot{q}_\sigma = \ddot{q}_\sigma = 0$
at $q_\sigma = q_\sigma^0$

Expand $q_\sigma \simeq q_\sigma^0 + \eta_\sigma$

Work to lowest order in η_σ

$$L \cong \frac{1}{2} \sum_{\sigma\lambda} \left(m_{\sigma\lambda} \dot{\eta}_\sigma \dot{\eta}_\lambda - V_{\sigma\lambda} \eta_\sigma \eta_\lambda \right)$$

$$m_{\sigma\lambda} = \sum_i m_i \left(\frac{\partial x_i}{\partial q_\sigma} \right)_{q^0} \left(\frac{\partial x_i}{\partial q_\lambda} \right)_{q^0} = m_{\lambda\sigma} = m^*_{\sigma\lambda}$$

$$V_{\sigma\lambda} \equiv \left(\frac{\partial^2 V}{\partial q_\sigma \partial q_\lambda} \right)_{q_0} = V_{\lambda\sigma} = V^*_{\sigma\lambda}$$

Normal Modes

$$\eta_\sigma = \operatorname{Re} z_\sigma \qquad z_\sigma = z^0_\sigma e^{i\omega t}$$

$$\det \left| V_{\sigma\lambda} - \omega^2 m_{\sigma\lambda} \right| = 0$$

$\omega^2 = \omega_s^2 \qquad s = 1, \ldots, n \qquad$ n normal mode frequencies

Eigenvector

$$\sum_\lambda \left[V_{\sigma\lambda} - \omega^2 m_{\sigma\lambda} \right] \rho^{(s)}_\lambda = 0$$

Normalization

$$\sum_{\lambda\sigma} \rho^{(t)}_\sigma m_{\sigma\lambda} \rho^{(s)}_\lambda = \delta_{st}$$

Modal matrix

$$A_{\lambda\sigma} \equiv \rho^{(\sigma)}_\lambda$$

Normal coordinates $\quad \eta = A\xi(t)$

Chapter 5 Rigid body motion

$$\left(\frac{dL}{dt}\right)_{\text{inertial}} = \Gamma^{(e)}$$

is valid if (1) origin is fixed in inertial frame or (2) about c.m.

Inertia tensor
Principal axes

Euler eq.
Euler angles