Lecture 2.6 Review

Midterm covers chapters 1-5
Short problems and some definitions

Chapter 1 Basic principles
Conserved quantities: $E$, $L$
Write $T = T' + T_{cm}$
and $L = L' + L_{cm}$
where $L_{cm} = R \times M \times R'$ etc.

Scattering cross section

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin \theta} \left| \frac{db}{d\sigma} \right|$$

$b =$ impact parameter, $\theta =$ scattering angle

Chapter 2 Accelerated coordinate systems

$$\left( \frac{d\vec{V}}{dt} \right)_{\text{inertial}} = \left( \frac{d\vec{V}}{dt} \right)_{\text{body}} + \vec{w} \times \vec{V}$$

Newton's laws in accelerated frame

$$m \left( \frac{d^2 \vec{r}}{dt^2} \right)_{\text{body}} = F'_{\text{coriolis}} - 2m \omega \times \left( \frac{d\vec{r}}{dt} \right)_{\text{body}} - m \omega \times (\omega \times \vec{r}) \quad \text{centripetal force}$$

$$- m \left( \frac{d\omega}{dt} \right) \vec{r} - m \left( \frac{d^2 a}{dt^2} \right)$$
Chapter 3 Lagrangian Dynamics

\[ L = T - V \]
\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0 \]

(i) Incorporate constraints and avoid dealing explicitly with forces of constraint.
(ii) Rewrite dynamics in any convenient generalized coordinates.

Review derivation of Lagrange's eq.
(a) From D'Alembert's principle \( \rightarrow \) Forces of constraint do no work under a virtual displacement.
(b) Minimize action using calculus of variations.

Definitions
- Holonomic constraints
- Generalized coordinates
- Virtual displacements
- D'Alembert's principle
- Hamilton's principle
- Forces of constraint
- Lagrange multipliers

\[ F(x_1, \ldots, x_n, t) = 0 \]
Symmetries and conserved quantities

Generalized momenta \( p_6 = \frac{\partial L}{\partial \dot{q}_6} \)

If \( \frac{\partial L}{\partial q_6} = 0 \) then \( q_6 \) is a cyclic coordinate

and \( \frac{d}{dt} p_6 = 0 \)

Hamiltonian \( H = \sum_6 p_6 \dot{q}_6 - L \)

If Lagrangian does not depend explicitly on time \( \frac{\partial L}{\partial t} = 0 \Rightarrow \frac{d}{dt} H = 0 \)

If potential is time-independent and constraints are time-independent \( \Rightarrow H = T + V = E \)

Chapter 4 Small Osc

Very general!

Static equilibrium \( -\frac{\partial V}{\partial q_6} = 0\) \( \Rightarrow q_6 = \dot{q}_6 = 0 \)

so \( \dot{q}_6 = \ddot{q}_6 = 0 \)

Expand \( q_6 = \dot{q}_6 + \eta_6 \)
Work to lowest order in $\eta$

$$L = \frac{1}{2} \sum \left( m_0 \eta_i \eta_i' - V_0 (\eta_i \eta_i') \right)$$

$$m_0 \chi = \sum m_i \frac{\partial}{\partial q_i} \left( \frac{\partial}{\partial q_i} \right) \chi = m_0 \chi = m_0 \chi$$

$$V_{\chi \chi} = \left| \frac{\partial^2 V}{\partial \chi \partial \chi} \right| = V_{\chi \chi} = V_{\chi \chi}$$

Normal Modes

$$\eta_0 = \Re Z_0$$

$$Z_0 = Z_0 e^{-i \omega t}$$

$$\det \left| V_{\chi \chi} - \omega^2 m_{\chi} \right| = 0$$

$$\omega^2 = \omega^2_s$$

$$s = 1, \ldots, n$$

Mode Frequencies

$$\sum \left[ V_{\chi \chi} - \omega^2 m_{\chi} \right] \rho_{\lambda}^{(s)} = 0$$

Normalization

$$\sum_{\chi} \rho_{\chi}^{(s)} m_{\chi} \rho_{\lambda}^{(s)} = \delta_{s \lambda}$$

Modal matrix

$$A_{\chi \sigma} \equiv \rho_{\chi}^{(s)}$$
Normal coordinates \( \eta = A \xi \eta \)

Chapter 5 Rigid body motion

\[
\left( \frac{dE}{dt} \right)_{\text{Inertial}} = \mathbf{P}(\theta)
\]

is valid if (1) origin is fixed in inertial frame or (2) about cm.

- Inertia tensor
- Principal axes
- Euler eq.
- Euler angles