10/22/00

Lecture 25 Obliquity and Climate of Mars

$$\boxed{L = \tfrac{1}{2} I_1 (\dot{\alpha}^2 \sin^2\beta + \dot{\beta}^2) + \tfrac{1}{2} I_3 (\dot{\alpha}\cos\beta + \dot{\gamma})^2 + \lambda \sin^2\alpha \sin^2\beta}$$

$$\lambda = -\frac{3}{2}\frac{G M_\odot}{r^3}(I_3 - I_1)$$

(β) $I_1 \ddot{\beta} - I_1 \dot{\alpha}^2 \sin\beta\cos\beta - I_3(\dot{\alpha}\cos\beta + \dot{\gamma})\dot{\alpha}\sin\beta - 2\lambda \sin\beta\cos\beta \sin^2\alpha = 0$

(α) $\frac{d}{dt}\left[I_1 \dot{\alpha}\sin^2\beta + I_3(\dot{\alpha}\cos\beta + \dot{\gamma})\cos\beta\right] - 2\lambda \sin^2\beta \cos\alpha \sin\alpha = 0$

(γ) $\frac{d}{dt} P_\gamma = 0 \qquad P_\gamma = I_3(\dot{\alpha}\cos\beta + \dot{\gamma}) = \text{const.}$

1. Expand around $\dot{\alpha} = \dot{\beta} = 0$, $\beta = \beta_0 + \eta_\beta$ to first order in $\dot{\alpha}, \dot{\beta}, \eta_\beta$ and λ
2. Average over position of Sun $\langle \sin^2 x \rangle = \frac{1}{2}$ $\langle \sin\alpha\cos\alpha \rangle = 0$

Note, perturbation theory $\Rightarrow$ expect $\dot{\alpha}, \dot{\beta}$ to be of order λ

$$I_1 \ddot{\eta}_\beta - P_\gamma \dot{\alpha} \sin\beta_0 = \lambda \sin\beta_0 \cos\beta_0$$

$$I_1 \ddot{\alpha} \sin^2\beta_0 + P_\gamma \frac{d}{dt}\left[\cos\beta_0 - \sin\beta_0 \eta_\beta\right] = 0$$

$$\dot{\eta}_\beta = \frac{I_1 \sin\beta_0}{P_\gamma}\ddot{\alpha} \longrightarrow \ddot{\eta}_\beta = \frac{I_1 \sin\beta_0}{P_\gamma}\dddot{\alpha}$$

$$\frac{I_1^2 \sin\beta_0}{P_\gamma}\dddot{\alpha} - P_\gamma \dot{\alpha}\sin\beta_0 = \lambda \sin\beta_0\cos\beta_0$$

$$\Rightarrow \boxed{\dot{\alpha} = -\frac{\lambda\cos\beta_0}{P_\gamma}} \qquad \dddot{\alpha} \simeq 0$$

$$\alpha(t) = \dot{\alpha}\, t + \alpha_0$$

$$P_\gamma = I_3 \omega \qquad \lambda = -\frac{3}{2}\frac{GM_\odot}{r^3}(I_3 - I_1)$$

Add contribution of Moon $\frac{M_\odot}{r^3} \rightarrow \frac{M_m}{r_{em}^3}$

$$\dot{\alpha} = \frac{3}{2}\left(\frac{G}{\omega}\right)\left(\frac{I_3 - I_1}{I_3}\right)\left[\frac{M_\odot}{r^3}\cos\beta_0 + \frac{M_m}{r_{em}^3}\langle\cos\beta_m\rangle\right]$$

$$\langle\cos\beta_m\rangle \simeq \cos\beta_0$$

approx. neglected eccentricity and inclination of Moon's orbit

Numbers $\omega = 7.29 \times 10^{-5}\ s^{-1}$ $\qquad \frac{I_3 - I_1}{I_3} = \frac{1}{305.3}$

$r_{se} = 1.50 \times 10^{13}$ cm

$M_\odot = 1.99 \times 10^{33}$ g

$G = 6.67 \times 10^{-8}$ 1(cgs)

$M_m = 7.4 \times 10^{25}$ g

$r_{em} = 3.9 \times 10^{10}$ cm $\qquad \beta_m \simeq \beta_0 = 23.5^\circ$

$$\dot{\alpha} = \frac{3}{2}\left[\frac{6.67 \times 10^{-8}}{7.29 \times 10^{-5}}\right]\left[\frac{1}{305.3}\right]\left[\frac{1.99 \times 10^{33}}{(1.50 \times 10^{13})^3} + \frac{7.4 \times 10^{22}}{(3.9 \times 10^{10})^3}\right]$$

$\cos 23.5^\circ$

$$\dot{\alpha} \simeq 7.6 \times 10^{-12}\ s^{-1}$$

$$\boxed{T = \frac{2\pi}{\dot{\alpha}} \simeq 26{,}000 \text{ Years}}$$

Notes

1. Most accurate $(I_3 - I_1)/I_3$ from measured $\dot{\alpha}$. However can estimate I_3 from model of Earth or just use $I_3 \simeq .4\,MR^2$ close to $.33\,MR^2$. Newton made a good estimate and first explained precession of Equinoxes

(2) Precessin rate for Mars

a) No big Moon! decreases rate $\sim 1/3$

b) $r = 1.52$ AU and $\dot{\alpha} \propto 1/r^3$

c) day $\sim$ same as Earth, $\beta_0 \approx 25.2^\circ$

d) $J_2 = (I_3 - I_1)/MR^2 = .00196$ Mars

$.00108$ Earth

From tracking two small Moons in 19^{th} cent.

e) I_3 Not exactly known

Best model $I_3 \approx 0.365\ MR^2$

compared to $0.33\ MR^2$ Earth

$\Rightarrow (I_3 - I_1)/I_3 \sim \frac{1}{186}$ $\left(\frac{1}{305} \text{ Earth} \right)$

$\dot{\alpha} = 1.2 \times 10^{-12}\ s^{-1}$ $\boxed{P \approx 170{,}000 \text{ Years}}$

(3) For an elliptical orbit or for Peturbation from another planet

$$\langle \lambda \sin\alpha \cos\alpha \rangle \neq 0$$

Time average may not be zero exactly because $\lambda \propto \frac{1}{r^3}$ and r also depends on time. Go back to α equation

$$\frac{d}{dt} p_\alpha = 2\sin^2\beta_0 \langle \lambda \sin\alpha \cos\alpha \rangle$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\alpha}} = I_1 \dot{\alpha} \sin^2\beta + p_\gamma \cos\beta \qquad \dot{\alpha} \approx -\frac{\lambda \cos\beta_0}{p_\gamma}$$

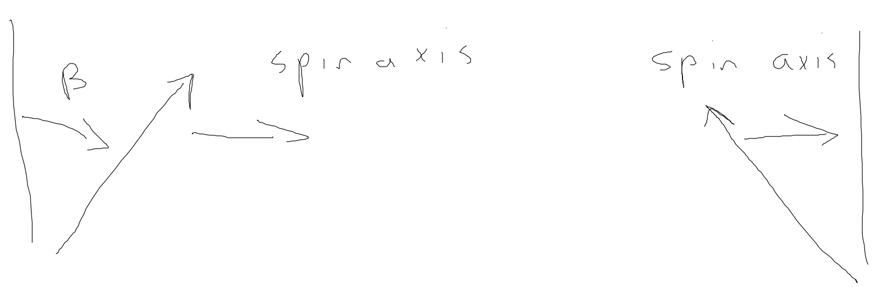
$$\boxed{\frac{d}{dt} \cos\beta = \frac{2\sin^2\beta_0}{p_\gamma} \langle \lambda \sin\alpha \cos\alpha \rangle}$$

This leads to changes in β = Obliquity
Which leads to large changes in climate

Petubations also change angle of inclination of orbit.

If inclination oscillates with same frequency as precession can have spin-orbit resonances and get large changes in β

If precession is fast compared to oscillations in inclination than most petubations on β will average to zero



petubation tends to increase β

13,000 years later petubation tends to decrease β

Petubation averages near zero over times long compared to 26,000 years

⇒ Large Moon protects obliquity of Earth and helps keep climate stable

Obliquity of Mars varied greatly over time with very large changes in climate. Strong evidence that Mars was warmer and wetter in past times.