10/13/00

Lecture 21 Precession of the Equinoxes

See for example Physics of the Earth, F. Stacey, John Wiley + Sons 1969

Torque from Sun + Moon on equatorial bulge of Earth causes direction of rotation to precess with period of 25,000 years.

Last time Multipole expansion

$$\frac{1}{|\vec{r}-\vec{r}'|} = \sum_{l=0}^{\infty} \frac{r_<^l}{r_>^{l+1}} P_l(\cos\theta_{rr'}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \frac{r_<^l}{r_>^{l+1}} \frac{4\pi}{2l+1} Y_{lm}^*(\theta,\phi)\, Y_{lm}(\theta',\phi')$$

$$\Phi(r,\theta,\phi) = -G\sum_{lm} \frac{1}{r^{l+1}}\left(\frac{4\pi}{2l+1}\right) Y_{lm}^*(\theta,\phi)\, Q_{lm}$$

$$Q_{lm} = \int d^3r'\, r'^l\, Y_{lm}(\theta',\phi')\, \rho(r',\theta',\phi')$$

ρ ← density of Earth

$l=1$ term vanishes if chose origin at cm

If ρ independent of ϕ → $m \neq 0$ terms vanish

$$\Phi \approx \Phi_0(r) + \Phi_2(r,\theta) + \dots$$

$$\Phi_0 = -\frac{GM_e}{r} \qquad \Phi_2 = -\frac{G}{r^3} P_2(\cos\theta)\, q_2$$

$$q_2 = \int d^3r'\, \rho(r')\, r'^2\, P_2(\cos\theta')$$

$$= \int d^3r'\, \rho(r')\left(\frac{3}{2}z'^2 - \frac{r'^2}{2}\right) \qquad P_2(x) = \frac{3x^2-1}{2}$$

$$I_3 = \int d^3r'\, \rho(r')\, (r'^2 - z'^2) \qquad I_1 = \int d^3r'\, \rho(r')\left(\frac{r'^2+z'^2}{2}\right)$$

$$\boxed{\Phi_2 = -\frac{G}{r^3}(I_1 - I_3)\, P_2(\cos\theta)}$$

$$\Phi = -\frac{G M_e}{r}\left[1 - J_2 \left(\frac{R_e}{r}\right)^2 P_2(\cos\theta)\right]$$

$$J_2 = (I_3 - I_1) / M_e R_e^2$$

R_e = average radius of earth

Note if spherical symmetry then $\Phi = -\frac{GM}{r}$ independent of radial distribution of mass

Gravity measurements can only tell you $I_3 - I_1$ can't get I_3 itself

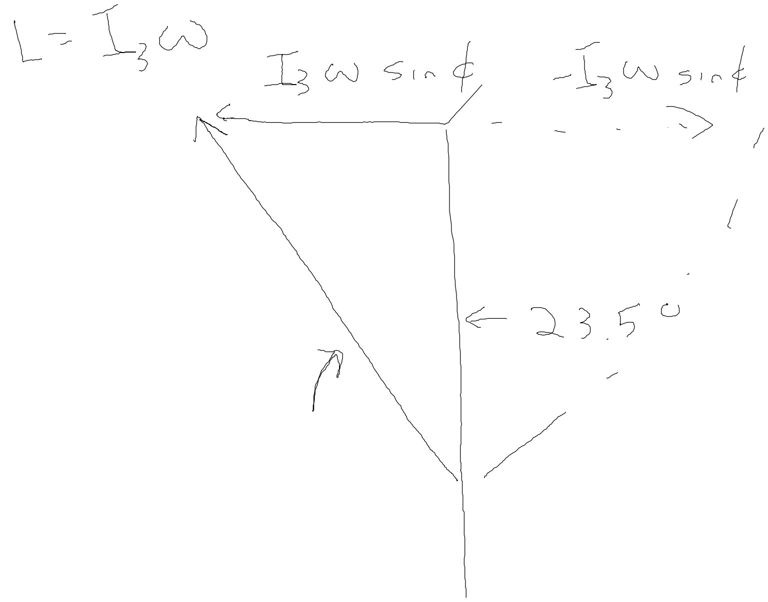
Because Φ depends on angle gravity can exert a nonzero torque on an object. For example Sun

$$\tau \sim -\frac{\partial \Phi}{\partial \theta} \propto -\frac{G M_e M_\odot}{r^3} J_2$$

Tide strength goes like $\frac{M_\odot}{r^3}$

By Newton 3 Sun exerts same torque back on Earth

This torque causes precession of Equinoxes



Torque causes axis to precess
In 13000 years it goes around 180°

$$\Delta L = 2 I_3 \omega \sin\phi = \langle \Gamma \rangle T$$

average torque

time

$$\frac{1}{T} \propto \frac{\Gamma}{I_3 \omega} \propto \frac{I_3 - I_1}{I_3}$$

Note total torque is sum of sun + moon + very small contributions from planets

Moon ~~to~~ bigger than sun $\frac{M_{moon}}{r_{em}^3} > \frac{M_\odot}{r_{es}^3}$
even though $\frac{M_\odot}{r_{es}} \gg \frac{M_{moon}}{r_{em}}$

Measure rate precession $\Rightarrow \frac{I_3 - I_1}{I_3} = \frac{1}{305.3}$

Note precession can be very important for climate. At the moment northern winter occurs when Earth is closest to Sun. Thus some cancelation between effects of tilt and distance. 13000 years ago northern winter occured when Earth was farthest from Sun.

Measure gravitational field by tracking spacecraft

$$J_2 = \frac{I_3 - I_1}{M_e R_e^2} = 1.083 \times 10^{-3}$$

$$\frac{I_3}{M_e R_e^2} = \left(\frac{I_3}{I_3 - I_1}\right)\left(\frac{I_3 - I_1}{M_e R_e^2}\right) = (305.3)\, 1.083 \times 10^{-3}$$

$$\boxed{I_3 = 0.33\, M_e R_e^2}$$

If Earth was a uniform sphere

$$I_3^U = 0.4\, M_e R_e^2$$

$$I_3 < I_3^U$$ implies mass concentrated towards center.

Thus measurement of I_3 proves Earth has a dense core.

Can get further information on density vs depth from seismic measurements.

Same measurements show Moon has very small core.

Apollo Moon rocks show crust of Moon very close in composition to crust of Earth. Less violatels such as hydrogen which can escape in Moon's gravity.

Core formed when dense iron seperated from less dense rocks because of gravity when Earth formed. Both Moon and Earth became hot enough to be liquid when formed.

⟹ Problem for origin of Moon.

If Moon was captured from far away in solar system why is crust like Earth.

If Moon formed nearby what happened to Moon's iron?

⟹ Solution Big Whack