10/2/00

Lecture 16 Normal Modes

Last time general solution

$$\eta_\lambda(t) = \sum_{s=1}^{n} c^{(s)} \rho_\lambda^{(s)} \cos(\omega_s t + \phi_s)$$

$\rho_\lambda^{(s)}$ is eigenvector with eigenvalue ω_s^2

$$(1) \quad \sum_\lambda [V_{\sigma\lambda} - \omega_s^2 m_{\sigma\lambda}] \rho_\lambda^{(s)} = 0$$

s tells which normal mode λ says ~~at~~ wich coordinate

Important <u>eigenvectors with different eigenvalues are orthogonal</u>

Write (1) for ω_t^2

$$(2) \quad \sum_\lambda (V_{\sigma\lambda} - \omega_t^2 m_{\sigma\lambda}) \rho_\lambda^{(t)} = 0$$

multiply (1) by $\rho_\sigma^{(t)}$ and sum on σ

$$(A) \quad \sum_{\sigma\lambda} \rho_\sigma^{(t)} V_{\sigma\lambda} \rho_\lambda^{(s)} - \omega_s^2 \rho_\sigma^{(t)} m_{\sigma\lambda} \rho_\lambda^{(s)} = 0$$

Multiply (2) by $\rho_\sigma^{(s)}$ and sum on σ

$$\sum_{\sigma\lambda} [\rho_\sigma^{(s)} V_{\sigma\lambda} \rho_\lambda^{(t)} - \omega_t^2 \rho_\sigma^{(s)} m_{\sigma\lambda} \rho_\lambda^{(t)}] = 0$$

Switch $\sigma \leftrightarrow \lambda$ $\quad V_{\sigma\lambda} = V_{\lambda\sigma} \quad m_{\sigma\lambda} = m_{\lambda\sigma}$

$$(B) \quad \sum_{\sigma\lambda} [\rho_\lambda^{(s)} V_{\sigma\lambda} \rho_\sigma^{(t)} - \omega_t^2 \rho_\lambda^{(s)} m_{\sigma\lambda} \rho_\sigma^{(t)}] = 0$$

Subtract B from A

$$(\omega_t^2 - \omega_s^2) \sum_{\sigma\lambda} \rho_\sigma^{(t)} m_{\sigma\lambda} \rho_\lambda^{(s)} = 0$$

If $\omega_t^2 \neq \omega_s^2 \Rightarrow$

$$\sum_{\sigma\lambda} \rho_\sigma^{(t)} m_{\sigma\lambda} \rho_\lambda^{(s)} = 0$$

or in Matrix notation

$$\underline{\rho}^{(t)\,T} \underline{\underline{m}} \, \underline{\rho}^{(s)} = 0$$

Eigenvector $\underline{\rho}^{(s)}$ is orthogonal (w.r.t. $\underline{\underline{m}}$) to $\underline{\rho}^{(t)}$ if $\omega_s^2 \neq \omega_t^2$

If $\omega_s^2 = \omega_t^2$ can chose normalization of $\rho_\lambda^{(s)}$

$$\sum_{\sigma\lambda} \rho_\sigma^{(s)} m_{\sigma\lambda} \rho_\lambda^{(s)} = 1$$

or

$$\underline{\rho}^{(s)\,T} \underline{\underline{m}} \, \underline{\rho}^{(s)} = 1$$

Thus eigen functions are orthonormal

$$\underline{\rho}^{(s)\,T} \underline{\underline{m}} \, \underline{\rho}^{(t)} = \delta_{st}$$

$$[\rho_1, \rho_2, \cdots \rho_n] \begin{bmatrix} & & \\ & n \times n & \\ & & \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix}$$