12/10/99

Lecture 39 Review

Chap. 1 Basic principles

Can seperate center of mass and internal motion

$$T = T_{cm} + T'$$

$$T_{cm} = \frac{1}{2} M V_{cm}^2 \qquad T' = \sum_i \frac{1}{2} m_i \dot{r}_i'^2$$

$$L = L_{cm} + L'$$

$$L_{cm} = M R \wedge V \qquad L' = \sum_i m_i r_i' \wedge v_i'$$

Central force motion $V(\vec{r}) = V(r)$

$$E = \frac{1}{2} m \dot{r}^2 + V_{eff}(r)$$

$$V_{eff} = V(r) + l^2 / 2mr^2$$

Inverse-square law gives elipses

$$E = -MmG/2a \qquad a = \text{semi major axis}$$

Chapter 2 Accelerated Coordinate Systems

$$\left(\frac{d}{dt}\right)_{\text{Inertial}} = \left(\frac{d}{dt}\right)_{\text{body}} + \omega \wedge$$

$$\left(\frac{dV}{dt}\right)_{\text{inert}} = \left(\frac{dV}{dt}\right)_{\text{body}} + \omega \wedge V$$

Chapter 3 Lagrangian Dynamics

Incorporate forces of constraint and use generalized coordinates

Holonomic constraints

$$f_j(x_1 \ldots x_n, t) = c_j \qquad j = 1, 2, \ldots k$$

$x_1 \ldots x_n$ = "real" coordinates

Generalized coordinates q_i

$$x_j = x_j(q_1 \ldots q_{n-k}, t) \qquad j = 1, \ldots n$$

Lagrangian

$$L = T - V = L(q_1 \ldots q_{n-k}, \dot{q}_1 \ldots \dot{q}_{n-k}, t)$$

Lagrange's eq.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\sigma} - \frac{\partial L}{\partial q_\sigma} = 0 \qquad \sigma = 1, \ldots, n-k$$

Hamilton's principle

$$\delta \int_{t_1}^{t_2} L[q(t), \dot{q}(t), t]\, dt = 0$$

$$\delta q(t_1) = \delta q(t_2) = 0$$

gives Lagrange's eq.

4 Small Oscillations

Expand about equil.

$$-\frac{\partial V}{\partial q_\sigma}\bigg|_{q_0} = 0 \qquad \sigma = 1, \dots, n$$

$$q_\sigma = q_\sigma^0 + \eta_\sigma$$

$$\dot{q}_\sigma = \dot{\eta}_\sigma$$

$$T = \frac{1}{2}\sum_{\sigma\lambda} m_{\sigma\lambda}\,\dot{\eta}_\sigma\,\dot{\eta}_\lambda$$

$$m_{\sigma\lambda} \equiv \sum_i m_i \left(\frac{\partial x_i}{\partial q_\sigma}\right)_{q^0}\left(\frac{\partial x_i}{\partial q_\lambda}\right)_{q_0}$$

$$V = V_0 + \frac{1}{2}\sum_{\sigma\lambda} V_{\sigma\lambda}\,\eta_\sigma\,\eta_\lambda$$

$$V_{\sigma\lambda} = \frac{\partial^2 V}{\partial q_\sigma \partial q_\lambda}\bigg|_{q_0}$$

$$\sum_\lambda \left(m_{\sigma\lambda}\,\ddot{\eta}_\lambda + V_{\sigma\lambda}\,\eta_\lambda\right) = 0 \qquad \sigma = 1, \dots, n$$

Normal modes

$$z_\sigma = z_\sigma^0\, e^{i\omega t}$$

$$\eta_\sigma = \mathrm{Re}\, z_\sigma$$

$$\sum_\lambda \left[V_{\sigma\lambda} - \omega^2 m_{\sigma\lambda}\right] z_\lambda^0 = 0$$

Eigenvalues

$$\det | V_{\sigma\lambda} - \omega^2 m_{\sigma\lambda} | = 0$$

eigenvectors

$$(\underline{V} - \omega_s^2 \underline{m}) \underline{\rho}^{(s)} = 0$$

$$\underline{\eta}(t) = \sum_s C^s \underline{\rho}^{(s)} \cos(\omega_s t + \phi_s)$$

Chap. 5 Rigid Bodies

$$T = \frac{1}{2} \sum_{ij=1}^{3} I_{ij} \omega_i \omega_j$$

$$L_i = \sum_j I_{ij} \omega_j$$

If body is not symmetric then $\vec{\omega}$ and $\vec{L}$ can point in different directions

$$I_{ij} = \int d^3r \, \rho(r) (\delta_{ij} r^2 - r_i r_j)$$

Hamiltonian Dynamics

$$p_\sigma = \frac{\partial L}{\partial \dot{q}_\sigma}$$

$$H = \sum_\sigma p_\sigma \dot{q}_\sigma - L$$

$L = L(q, \dot{q}, t)$

$H = H(p, q, t)$

$$\frac{\partial H}{\partial p_\sigma} = \dot{q}_\sigma$$

$$\frac{\partial H}{\partial q_\sigma} = -\dot{p}_\sigma$$

Lagrange's equations are invariant under transformation of generalized coordinates

Hamilton's equations are invariant under much broader class of

<u>Canonical transformations</u>

Transformation canonical if form of Hamilton's eq. is unchanged

For example can mix up coordinates and momenta

<u>Poisson Brackets</u>

$$[F, G]_{PB} = \sum_\sigma \left(\frac{\partial F}{\partial q_\sigma} \frac{\partial G}{\partial p_\sigma} - \frac{\partial F}{\partial p_\sigma} \frac{\partial G}{\partial q_\sigma} \right)$$

$$\frac{dF}{dt} = -[H, F]_{PB} + \frac{\partial F}{\partial t}$$

<u>Canonical quantization procedure</u>

In QB replace

$$[A, B]_{PB} \longrightarrow \frac{1}{i\hbar} [\hat{A}, \hat{B}]$$

$$\frac{d\hat{F}}{dt} = \frac{i}{\hbar} [\hat{H}, \hat{F}] + \partial \hat{F}/\partial t$$

Strings

$$L = \int_0^{\ell} \mathcal{L}\, dx$$

$$\mathcal{L} = \frac{1}{2}\sigma(x)\left(\frac{\partial u}{\partial t}\right)^2 - \frac{1}{2}\tau(x)\left(\frac{\partial u}{\partial x}\right)^2$$

Eigenfunctions satisfy sturm Liouville eq.

$$-\frac{\partial}{\partial x}\tau\frac{d\rho}{dx} + v(x)\rho = \omega^2\sigma\rho$$

Eigenfunctions are orthogonal

$$\int_a^b \rho_p(x)^* \rho_q(x)\, dm = \delta_{pq}$$

$$dm = \sigma(x)\, dx$$

$$u(x,t) = \rho(x)\cos(\omega t + \phi)$$

Eigenfunctions are complete

Variational Principle

$$\omega^2[\rho] = \frac{\frac{1}{2}\int_a^b \left[\tau(x)(d\rho/dx)^2 + v(x)\rho^2\right] dx}{\frac{1}{2}\int_a^b [\sigma(x)\rho^2]\, dx}$$

ok guess f_0, generates

good guess f_0, ω^2

$$\omega^2[\rho] \geq \omega^2[\rho_1]$$

Green's Functions

Consider inhomogeneous Sturm Liouville eq

$$-\frac{d}{dx} \tau \frac{\partial u}{dx} + v u - \omega^2 \sigma u = \sigma(x) f(x)$$

To calculate response of system to driving force $f(x)$

Introduce $(L_0 - \omega^2 \sigma) G_\omega(x,y) = \delta(x-y)$

$$u(x) = \int_a^b G_\omega(x, y') \sigma(y') f(y') \, dy'$$

$$G_\omega = \sum_{n=1}^{\infty} \frac{\rho_n(x) \rho_n(y)}{\omega_n^2 - \omega^2}$$

or

$$= - \frac{u_1(x_<) u_2(x_>)}{C}$$

Perturbation Theory

$$\rho_n(x) = \rho_n(x) + \epsilon \sum_{q \neq n} \rho_q(x) \frac{1}{\omega_n^2 - \omega_q^2} \langle q|v|n\rangle + O(\epsilon^2)$$

$$\omega^2 - \omega_n^2 = \epsilon \langle n|v|n\rangle + O(\epsilon^2)$$