

12/4/00

Lecture 41 Sound in the Sun

Last time

$$\rho = \rho_0 + \rho'$$

ρ_0 ← equilibrium density ρ' ← small perturbation

or $[\nabla^2 + k^2] \rho' = 0$

$$[\nabla^2 + k^2] \Phi = 0$$

$$\omega = kc$$

speed of sound

$$c^2 \equiv \left(\frac{\partial p}{\partial \rho} \right)_s$$

Velocity pot.

$$\vec{V} = -\vec{\nabla} \Phi$$

and

$$p = p_0 + p'$$

$$p' = \rho_0 \frac{\partial \Phi}{\partial t}$$

p_0 ← equilibrium pressure

B. Conditions

$$\hat{n} \cdot \vec{\nabla} \Phi = 0$$

$$\frac{\partial \Phi}{\partial t} = 0$$

Fixed wall
free surface

Speed of sound in ideal gas

Example Sun is nearly ideal plasma
of ions + electrons.

Ideal gas eq. of state

$$p = \frac{RT}{M_0} \rho$$

$$R = N_0 k$$

M_0 = molecular wt. N_0 = avogadro's #
in grams

ρ = density (grams/volume) $\equiv 1/V$

V = volume per gram

Internal energy (equ. partition theorem)

$$E = \frac{f}{2} \frac{RT}{M_0} \quad E = \text{Energy per gram}$$

$$f = \# \text{ of degrees of freedom} = 3 \text{ translations, } f_c, \text{ monatomic ideal gas}$$

1st Law of Thermo.

$$dE = dq + dw \quad \begin{array}{l} \text{Heat flow into system} \\ \text{Work done on system} \end{array}$$

$$dq = T ds = dE + p dV$$

$$ds = \frac{1}{T} \frac{f}{2} \frac{R}{M_0} dT + \frac{1}{T} \left(\frac{RT}{M_0 V} \right) dV$$

$$ds = \frac{R}{M_0} \left[\frac{f}{2} \frac{dT}{T} + \frac{dV}{V} \right]$$

$$dS = \frac{R}{M_0} d \ln [T^{f/2} V]$$

$$S = \frac{R}{M_0} \ln [T^{f/2} V] + \text{const.}$$

$$T = \frac{M}{R_0} pV$$

$$S = \frac{R}{M_0} \ln p^{f/2} V^{f/2 + 1} + c'$$

$$S = \frac{f}{2} \frac{R}{M_0} \ln p V^\gamma + c'$$

$$\gamma = \text{adiabatic index} = \frac{f+2}{f}$$

$$\gamma = C_p / C_v = 5/3 \quad \text{for monatomic ideal gas}$$

At constant S

$$P = P_0 V^{-\gamma} = P_0 \rho^{\gamma}$$

$$c^2 = \left(\frac{\partial P}{\partial \rho} \right)_S = \gamma \left(\frac{P}{\rho} \right) = \gamma \frac{RT}{M_0}$$

$$c = \gamma^{1/2} \left(\frac{RT}{M_0} \right)^{1/2}$$

Speed of sound for ideal gas grows with T . Also c depends on composition

Example pure atomic H $M_0 = 1$ gram/mole
 Hydrogen plasma $M_0 = 1/2$ gram/mole!
 1 gram = 1 mole of protons + 1 mole of electrons
 Pure He plasma 4 grams = 1 mole of ions
 2 moles of electrons

$$M_0 = 4/3 \text{ gram/mole}$$

Note Sun is 27% He by mass originally

Temperature of Sun increases with decreasing radius. Therefore sound speed also increases

If $c^2(T(r))$ depends on position

$$\left[\nabla \cdot c^2(r) \nabla + \omega^2 \right] p'(r) = 0$$

This causes sound waves to bend back towards the surface.

Standard Solar Model

See for example J.N. Bahcall, "Neutrino Astrophysics" chapters 2, 4

Model of Sun important for

① Solar neutrinos: Flux of neutrinos sensitive function of central temperature

We detect fewer solar ν than expected.
Probably not lower central temp.
(i) Helioseismology sound speeds agree with observations

(ii) Spectrum of ν energies observed is not consistent with slightly cooler Sun.
Solar ν problem is probably ν osc.

$$\nu_e \rightarrow \nu_\mu \text{ or } \nu_\tau \text{ or } \nu_s$$

② Understanding of stellar evolution and the synthesis of the chemical elements.
Use standard solar model as an important check of our models for other mass stars.

Note asteroseismology \rightarrow in some cases possible to observe low l modes of other stars.

$$[\nabla^2 + k^2] \rho'(\vec{r}) = 0$$

Normal modes

$$\rho'(\vec{r}) = R_{nl}(r) Y_l^m(\theta, \phi)$$

Don't have spatial resolution to observe high l modes for other stars

③ Detailed understanding of Sun important for Earth's climate and life on Earth.

Example of all the more it we don't understand, origin of 11 year solar cycle. However in time with detailed observations we may.

Theory predicts Sun 4 billion years ago (when life started on Earth) was 30% less luminous. This has important implications for Earth's and Mars' early climate.

Hydrostatic model of Sun

Pressure rises because of weight of material above you

$$\frac{dP}{dr} = - \frac{G M(r) \rho(r)}{r^2}$$

$$M(r) = 4\pi \int_0^r r'^2 dr' \rho(r') = \text{Mass enclosed}$$

$$M(0) = 0 \quad M(R) = M_{\odot}$$

Energy transport (We did not discuss)

$$L_r = - 4\pi r^2 \frac{ac^3}{3} \frac{1}{\kappa \rho} \frac{dT}{dr}$$

L_r = luminosity at distance r

κ = opacity

a = stephane - Boltzmann const.

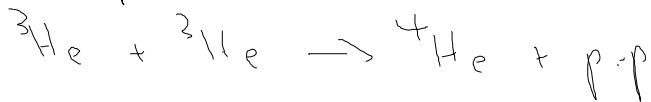
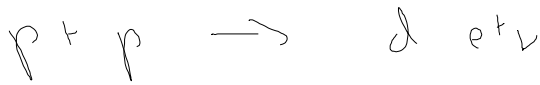
" c " = speed of light

Opacity = resistance to radiation flow from photons scattering off of electrons 5

ρ is capacity gradient needed implies large temperature to drive solar luminosity

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon_{\text{nuclear}}(\rho, T, \text{comp.})$$

$\epsilon_{\text{nuclear}}$ = rate of energy generation from nuclear reactions.



$\epsilon_{\text{nuclear}}$ of T is somewhat sensitive function

Also have equation of state

$$P = P(T, \rho, \text{comp.})$$

which is just ideal gas + corrections

(A) Guess central density and temperature

(B) Integrate $\frac{dP}{dr}$, $M(r)$, L_r outwards till $P=0$

This gives L_0 , R , $M_0 = M(R)$

(C) adjust initial conditions in (A) until output agrees with measured values.