12/4/00

Lecture 41 Sound in the Sun

Last time $\rho = \rho_0 + \rho'$ ← small perturbation; ρ_0 ← equilibrium density

$$[\nabla^2 + k^2]\,\rho' = 0$$

or

$$[\nabla^2 + k^2]\,\Phi = 0 \qquad \omega = kc$$

speed of sound $\quad c^2 \equiv \left(\dfrac{\partial p}{\partial \rho}\right)_s$

Velocity pot. $\quad \vec{v} = -\vec{\nabla}\Phi$

and

$$p = p_0 + p' \qquad p' = \rho_0 \frac{\partial \Phi}{\partial t}$$

p_0 ← equilibrium pressure

B. Conditions

$\hat{n}\cdot\vec{\nabla}\Phi = 0$ fixed wall

$\dfrac{\partial \Phi}{\partial t} = 0$ free surface

Speed of sound in ideal gas

Example Sun is nearly ideal plasma of ions + electrons.

Ideal gas eq. of state

$$p = \frac{RT}{M_0}\rho$$

$R = N_0 k \qquad N_0 =$ avogadro's #

$M_0 =$ molecular wt. in grams

$\rho =$ density (grams/volume) $\equiv 1/V$

$V =$ volume per gram

Internal energy (equ. partition theorem)

$$\epsilon = \frac{f}{2} \frac{RT}{m_0}$$ ϵ = energy per gram

f = # of degrees of freedom
= 3 translations for monoatomic ideal gas

1st Law of Thermo.

$$d\epsilon = dq + dw$$ work done on system

heat flow into system

$$dq = T ds = d\epsilon + p\, dV$$

$$ds = \frac{1}{T} \frac{f}{2} \frac{R}{m_0} dT + \frac{1}{T}\left(\frac{RT}{m_0 V}\right) dV$$

$$dS = \frac{R}{m_0}\left[\frac{f}{2}\frac{dT}{T} + \frac{dV}{V}\right]$$

$$dS = \frac{R}{m_0} d\ln\left[T^{f/2} V\right]$$

$$S = \frac{R}{m_0} \ln\left[T^{f/2} V\right] + \text{const.}$$

$$T = \frac{m}{R_0} pV$$

$$S = \frac{R}{m_0} \ln p^{f/2} V^{\frac{f}{2}+1} + c'$$

$$S = \frac{f}{2} \frac{R}{m_0} \ln pV^{\gamma} + c'$$

$$\gamma = \text{adiabatic index} = \frac{f+2}{f}$$

$$\gamma = C_p / C_v = 5/3$$ for monoatomic ideal gas

At constant s

$$p = p_0 V^{-\gamma} = p_0 \rho^{\gamma}$$

So

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = \gamma \left(\frac{p}{\rho}\right) = \gamma \frac{RT}{m_0}$$

$$\boxed{c = \gamma^{1/2} \left(\frac{RT}{m_0}\right)^{1/2}}$$

Speed of sound for ideal gas grows with T. Also c depends on composition

Example Pure atomic H Hydrogen plasma $m_0 = 1$ gram/mole / $m_0 = \frac{1}{2}$ gram/mole
1 gram = 1 mole of protons + 1 mole of electrons

Pure ^4He plasma 4 grams = 1 mole of ions + 2 moles of electrons

$$m_0 = 4/3 \text{ gram/mole}$$

Note Sun $\approx$ 27% He by mass originally

Temperature of Sun increases with decreasing radius. Therefore sound speed also increases

If $c^2(T(r))$ depends on position

$$[\vec{\nabla} \cdot c^2(r) \vec{\nabla} + \omega^2] \rho'(r) = 0$$

This causes sound waves to bend back towards the surface.

Standard Solar Model

See for example J.N. Bahcall, "Neutrino Astrophysics" Chapters 2, 4

Model of Sun important for

(1) Solar neutrinos: Flux of neutrinos sensitive function of central temperature

We detect fewer solar ν than expected. Probably not lower central temp
(i) Helioseismology sound speeds agree with observations
(ii) Spectrum of ν energies observed is not consistent with slightly cooler Sun.
Solar ν problem is probably ν osc.

$$\nu_e \longrightarrow \nu_\mu \text{ or } \nu_\tau \text{ or } \nu_s$$

(2) Understanding of stellar evolution and the synthesis of the chemical elements. Use standard solar model as an important check of our models for other mass stars.

Note asteroseismology → in some cases possible to observe low l modes of other stars.

$$[\nabla^2 + k^2]\, \rho'(\vec{r}) = 0$$

Normal modes

$$\rho'(\vec{r}) = R_{nl}(r)\, Y_l^m(\theta, \phi)$$

Don't have spatial resolution to observe high l modes for other stars

(3) Detailed understanding of Sun important for Earth's climate and life on Earth.

Example at the moment we don't understand origin of 11 year solar cycle. However in time with detailed observations we may.

Theory predicts sun 4 billion years ago (when life started on Earth) was ~ 30% less luminous. This has important implications for Earth's and Mars's early climate.

Hydrostatic model of Sun

Pressure rises because of weight of material above you

$$\frac{dP}{dr} = -\frac{G\,M(r)}{r^2}\,\rho(r)$$

$$M(r) = 4\pi \int_0^r r'^2\,dr'\,\rho(r') = \text{Mass enclosed}$$

$$M(0) = 0 \qquad M(R) = M_\odot$$

Energy transport (We did not discuss)

$$L_r = -4\pi r^2 \frac{a\text{"}c\text{"}}{3} \frac{1}{\kappa\rho} \frac{dT^4}{dr}$$

L_r = Luminosity at distance r
κ = opacity
a = Stephan-Boltzmann const.
"c" = speed of light

Opacity = resistance to radiation flow from photons scattering off of electrons

Big opacity implies large temperature gradient needed to drive solar luminosity

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \, \epsilon_{nuclear}(\rho, T, \text{comp.})$$

$\epsilon_{nuclear}$ = rate of energy generation from nuclear reactions.

$$p + p \rightarrow d\, e^+ \nu$$

$$d + p \rightarrow {}^3He + \gamma$$

$${}^3He + {}^3He \rightarrow {}^4He + p + p$$

Net $4p \rightarrow {}^4He + 2\nu_e + 2e^+$

$\epsilon_{nuclear}$ is somewhat sensitive function of T

Also have equation of state

$$P = P(T, \rho, \text{comp.})$$

which is just ideal gas + corrections

(A) Guess central density and temperature

(B) Integrate $\frac{dP}{dr}$, $M(r)$, L_r outwards till $P = 0$

This gives $L_\odot$, R, $M_\odot = M(R)$

(C) adjust initial conditions in (A) until output agrees with measured values.