P521 Lector 2 8/30/00

Read Chapter 1 F+W
Homework #1 Out Monday 9/4 Due Monday 9/11
No class 9/22 Friday

Conservation Laws

Can work directly with Newton's laws but many advantages to define linear and angular momentum, energy ...

Linear momentum

For any closed system (no outside forces) N_3 implies

$$\frac{d}{dt}(\vec{P}_{tot}) = 0 = \sum_{i,j} F_{ij} = \frac{1}{2}\sum_{i,j}(F_{ij}+F_{ji}) = 0$$

$$P_{tot} = \sum_i p_i \qquad \frac{dp_i}{dt} = F_i^{tot} = \sum_j F_{ij}$$

Define $F_{ii} = 0$

Angular momentum

$$\vec{L} = \vec{r}\wedge\vec{p} \qquad (r \wedge p \text{ or } r\times p)$$

$$\dot{L} = \vec{r}\wedge\vec{F} = \Gamma \quad \text{(torque)}$$

$$= \underbrace{\cancel{\dot{r}\wedge p}}_{0} + r\wedge\dot{p}$$

Energy + Work

Consider a static force field $\vec{F}(\vec{r})$

Work to move test particle from 1 to 2

$$W_{1\to 2} = \int_1^2 d\vec{s} \cdot \vec{F}(\vec{r})$$

along path s $\qquad \vec{F} = m \frac{d\vec{V}}{dt}$

$$W_{12} = \int ds \cdot m \frac{dV}{dt} = \int_1^2 dt\, \vec{V} \cdot m \frac{dV}{dt}$$

$$= m \int dt \frac{d}{dt}\left(\tfrac{1}{2}\vec{V}^2\right) = \tfrac{1}{2} m \left(V_2^2 - V_1^2\right)$$

independent of path. Define $T = \frac{1}{2} m V^2$

$$W_{12} = T_2 - T_1$$

If F has the special form

$$\vec{F} = -\vec{\nabla} U$$

where U is potential. Such forces are conservative

$$W_{1\to 2} = T_2 - T_1 = -\int_1^2 d\vec{s} \cdot \nabla U = -\int_1^2 dU$$

$$= U_1 - U_2$$

so

$$T_1 + U_1 = T_2 + U_2$$

For conservative forces $E = T + U$ is conserved.

System of Particles

Seperation of relative and center of mass quantities

$$\vec{R} \equiv M^{-1} \sum_{i=1}^{N} m_i \vec{r}_i \qquad M = \sum_{i=1}^{N} m_i$$

Let there be external forces F_i^e on each particle and relative forces F_{ij} between particles

$$\dot{p}_i = F_i^e + \sum_{j \neq i} \vec{F}_{ji}$$

define $F_{ii} = 0$

$$M \ddot{R} = \sum m_i \ddot{r}_i = \sum_i \dot{p}_i = \sum_i F_i^e + \sum_i \sum_j{}' F_{ji}$$

but

$$\sum_i \sum_j F_{ji} = \frac{1}{2} \sum_{ij} (F_{ji} + F_{ij}) = 0 \text{ by N3}$$

switch dummy variables $i \to j'$ $j \to i'$

$$\sum_{i=1}^{N} \sum_{j=1}^{N} F_{ij} = \sum_{j'=1}^{N} \sum_{i'=1}^{N} F_{j'i'} = \sum_{i=1}^{N} \sum_{j=1}^{N} F_{ji}$$

because N3 says $F_{ji} = -F_{ij}$

$$M \ddot{R} = M \dot{V} = \sum_i F_i^e = F_{tot}^e$$

Center of mass motion indep. of interactions between particles. It depends only on the total external force

Angular Momentum

$$\vec{L} = \sum_i \vec{r}_i \wedge \vec{p}_i = \sum_i L_i$$

$$\dot{L} = \cancel{\sum_i \dot{r}_i \wedge p_i} + \sum_i r_i \wedge \dot{p}_i = \sum_i r_i \wedge \left(F_i^e + \sum_j F_{ji}\right)$$

$$\sum_{i,j=1}^{N} r_i \wedge F_{ji} = \frac{1}{2} \sum_{i,j} \left(r_i \wedge F_{ji} + r_j \wedge \underbrace{F_{ij}}_{-F_{ji}}\right)$$

$$= \frac{1}{2} \sum_{i,j} (r_i - r_j) \wedge F_{ji}$$

If F_{ji} is a central force

it is directed along $\vec{r_i - r_j}$

$$\vec{F}_{ji}(\vec{r_i - r_j}) \propto \vec{r_i - r_j} \quad \text{for central forces}$$

$$(\vec{r_i - r_j}) \wedge (\vec{r_i - r_j}) \equiv 0$$

$$\dot{L} = \sum_i (r_i \wedge F_i^e) \equiv \Gamma^e \quad \text{external torque}$$

Relative + CM Coordinates

$$\vec{r}_i = \vec{R} + \vec{r}_i' \quad \text{or} \quad \vec{r}_i' = r_i - R$$

CM relative

$$v_i = V + v_i' \qquad V \equiv \dot{R}$$

$\sum_i m_i r_i' = 0$ for all time so

$$\sum_i m_i \dot{r}_i' = 0 = \sum m_i v_i'$$

$$L = \sum_i r_i \wedge p_i = \sum_{i=1}^{N} (\overrightarrow{R + r_i'}) \wedge m_i (V + v_i')$$

$$= R \wedge V \sum_i m_i + \sum_i \vec{r}_i' \wedge m_i v_i'$$

$$+ R \wedge \sum_i m_i v_i' + \sum_i r_i' \wedge m_i V$$

$$= 0 \qquad \sum_i m_i r_i' = 0$$

$$L = \vec{R} \wedge M\vec{V} + \underbrace{\sum_{i=1}^{N} r_i' \wedge m_i v_i'}_{L'}$$

$$\vec{L} = \vec{L}_{cm} + \vec{L}'$$

$$L_{cm} = \vec{R} \wedge M\vec{V}$$

Also works for energy

$$V_{tot} = \sum_i V^e(r_i) + \frac{1}{2} \sum_{ij} V(r_{ij})$$

$$F_i^e = -\nabla_i V^e(r_i) \qquad F_{ij} = -\nabla V r_{ij}$$

$$T = \frac{1}{2} \sum m_i v_i^2 = T_{cm} + T'$$

$$T' = \frac{1}{2} \sum_i m_i v_i'^2 \qquad T_{cm} = \frac{1}{2} M V^2$$

$$E = E_{cm} + E' = T + V$$

$$E_{cm} = T_{cm} + V_{cm}$$

$$E' = T' + V'$$