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P521 Lecture #1

Introduction Course Organization and Newton's laws

Organization: Chuck Horowitz
office Swain West 233
Email: Charlie@iucf.indiana.edu
Horowit@indiana.edu
Web: physics.indiana.edu/~charlie/cm

Text: Fetter + Walecka "Theoretical Mechanics of Particles and Continua"

Grade: Homework
Midterm
Final

Note, no late homework!

Grader: Si-yeon Kim

Goals: Gain a firm working knowledge of Classical mech. Most of you have seen much of this before now is your chance to make sure you understand it in depth.

Understand the classical field theory for quantum field theory.

Outline

Questions

Contact info.

Newton's laws

Easy to state
Profound implications
Central to all subsequent work.

First define a primary inertial coordinate system that is at rest with respect to the fixed stars. Now we could use "rest frame" of microwave background

1st law: In this primary inertial frame, every body remains at rest or in uniform motion unless acted on by a force $\vec{F}$. The condition $\vec{F} = 0$ thus implies a constant velocity v and a constant momentum $mv = p$

Newton's first law asserts that such an inertial frame exists. Clearly the first law is not a special case of the second law $F = ma$ with $F = 0 \Rightarrow a = 0$ because the first law is needed to define an inertial frame in which the 2nd law holds

2nd law: In the primary inertial frame application of a force alters the momentum

$$\vec{F} = \frac{d\vec{p}}{dt} = \dot{\vec{p}}$$

3rd law: To each action, equal and opposite reaction

$$\vec{F}_{12} = -\vec{F}_{21}$$

Is N2 true?
a) Of course not. At high velocities or for small systems it needs to be modified

b) Of course it is. There are now so many applications and observations that support it. Most importantly N2 helps define the force. An unexpected acceleration observed for same system would not be interpreted as a failure of N2 but rather a new force.

See for example J. D. Anderson et al, PRL 81 ('98) 2858 "The Apparent Anomalous, Weak, Long-Range Acceleration of Pioneer 10 and 11"

Newton's laws say very little about forces except that they conserve momentum $F_{12} = -F_{21}$

Newton's laws help to define a frame work for analyzing motions. Rather then give up the frame work we could define new forces.

So rather then asking is N2 true or not ask is this frame work usefull and the answer is clearly yes!

N Laws are usefull if the forces are "simple"

Example Gravity $\vec{F}_{21} = -G m_1 m_2 \frac{\overrightarrow{r_1 - r_2}}{|\overrightarrow{r_1 - r_2}|^3}$

Coulomb's law $\vec{F}_{21} = Q_1 Q_2 \frac{\overrightarrow{r_1 - r_2}}{|\overrightarrow{r_1 - r_2}|^3}$

Striking that both are inverse square laws

Galilean relativity

Any frame moving with constant velocity is again inertial. Inertial $\iff$ N1 holds

Proof: let $\vec{r}$ $\vec{r}'$ be coordinates as seen in two different frames moving with velocity $\vec{V}$

$$\vec{r}' = \vec{r} + \vec{V}t$$

$$\vec{r}_i - \vec{r}_j = \vec{r}_i' - \vec{r}_j'$$

So if forces only depend on $\vec{r}_{ij}$ then

$$\vec{F}_{ij}' = \vec{F}_{ij}$$ forces are same in two frames

then $$\frac{d^2\vec{r}}{dt^2} = \frac{d^2\vec{r}'}{dt^2}$$

So if
$$F = m\frac{d^2\vec{r}}{dt^2}$$
then
$$F' = m\frac{d^2\vec{r}'}{dt^2}$$

Thus N2 and N3 hold in any inertial frame

Conservation Laws

Can work directly from N laws but many advantages to define linear and angular momentum, energy ... which satisfy simple relations

<u>Linear momentum</u>: Whenever force vanishes momentum is conserved.

for any closed system (no outside forces) then N3 implies

$$\frac{d}{dt}\left(\vec{P}_{tot} = \sum_i \vec{p}_i\right) = 0$$

<u>Angular momentum</u> $L = \vec{r} \times \vec{p}$

Note sometimes I say $\vec{r} \wedge \vec{p} \equiv \vec{r} \times \vec{p}$

$$\dot{L} = \vec{r} \wedge \vec{F} \equiv \Gamma \quad \text{(torque)}$$

<u>Energy and Work</u>

Consider a static force field $\vec{F}(\vec{r})$

Work to move a test particle from 1 to 2 is

$$W_{1\to 2} = \int_1^2 d\vec{s} \cdot \vec{F}(\vec{r})$$

along path s, $\vec{F} = m\frac{d\vec{V}}{dt}$

$$W_{12} = \int_1^2 ds \cdot m\frac{dV}{dt} = \int_1^2 dt\, \vec{V} \cdot m \frac{d\vec{V}}{dt}$$

$$= m \int dt \frac{d}{dt}\left(\tfrac{1}{2}V^2\right) = \tfrac{1}{2}m\left(V_2^2 - V_1^2\right)$$

independent of the path. If $T \equiv \frac{1}{2}mV^2$ denotes the kinetic energy. The work done is $T_2 - T_1$

If F has the special form

$$\vec{F} = -\vec{\nabla}\, U(\vec{r})$$

where U is the potential. Such forces are conservative (although common they are quite restrictive)

$$W_{1\to 2} = T_2 - T_1 = -\int_1^2 d\vec{s}\cdot\vec{\nabla} U = -\int_1^2 dU$$
$$= U_1 - U_2$$

So for conservative forces

$$T_1 + U_1 = T_2 + U_2$$

or $E = T + V$ is conserved for conservative forces.

Note two equivalent def. of conservative forces

$$\nabla \wedge \vec{F} = 0 \quad \text{for all } r$$

$$\oint ds \cdot F = 0 \quad \text{for all closed paths}$$

{so work is indep. of path}