P521 Lecture #1

Introduction Course Organization and Newton's laws

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Text: Ferrer & Walecka, "Theoretical Mechanics of Particles and Continua"

Grades: Homework Midterm Final

Note: no late homework!

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Goals: Gain a firm working knowledge of classical mech. Most of you have seen much of this before, now is your chance to make sure you understand it in depth.

Understand the classical field theory for quantum field theory.

Outline

Questions

Contact info.
Newton's laws

Easy to state

Profound implications

Central to all subsequent work.

First, define a primary inertial coordinate
system s' = a rest frame with respect to
the fixed stars. Now we could use
"rest frame" or microwave background

1st law. In this primary inertial frame,
everybody remains at rest or in uniform
motion unless acted on by a force F. The
condition \( \mathbf{F} = 0 \) thus implies a constant
velocity \( \mathbf{V} \) and a constant momentum \( \mathbf{MV} \).

Newton's first law asserts that such an
inertial frame exists. Clearly, his first
law is not a special case of his
second law, \( \mathbf{F} = m \mathbf{a} \), because the first law is needed to define
an inertial frame in which the 2nd law
holds.

2nd law: In the primary inertial frame, application
of a force \( \mathbf{F} \) alters the momentum
\( \mathbf{p} \) according to
\[
\frac{d\mathbf{p}}{dt} = \mathbf{F}
\]

3rd law: To each action, equal and opposite
reaction \( \mathbf{F}_2 = -\mathbf{F}_1 \).

Is \( \text{N}2 \) true?

Or, of course not. At high velocities for small systems it needs to be modified.
Squaring the ball is not intuitive. Square laws are:

\( F = \frac{m \cdot a}{L} \)

Example:

Inertial force:

\( F = 0 \cdot \frac{m \cdot a}{L} = 0 \)

Thus, the law is useless. An experiment was conducted to determine the effect of force on mass. The result was:

\( F = m \cdot a \)

A very similar result was obtained, but in the first case, the force was found to be very small, about 2.5 Newtons. An exception is that it is considered very likely that Newton's law holds in a 2-dimensional system. An example is the calculation of the force on a Hooke's law spring.


Long, 1982. A different take on the same system.
Galilean relativity

Any frame moving with constant velocity is again inertial. Inertial \( \Leftrightarrow \) N1 holds.

Proof: Let \( F \) be coordinates as seen in two different frames moving with velocity \( \mathbf{V} \):

\[ F' = F + \mathbf{V} t \]

\[ F_i - F_j = F'_i - F'_j \]

So if forces only depend on \( F_{ij} \) then

\[ F'_{ij} = F_{ij} \]

forces are same.

Then

\[ \frac{d^2 F}{dt^2} = \frac{d^2 F'}{dt^2} \]

So if

\[ F = m \frac{d^2 \mathbf{r}}{dt^2} \]

then

\[ F' = m \frac{d^2 \mathbf{r}'}{dt^2} \]

Thus N2 and N3 hold in any inertial frame.

Conservation Laws

Can work directly from N laws but many advantages to define linear and angular momentum, energy ... which satisfy simple relations.
for any closed system (no outside forces)

\[
\frac{d}{dt} \mathbf{P}_{\text{tot}} = \Sigma \mathbf{F}_i
\]

Angular momentum \( \mathbf{L} = \mathbf{r} \times \mathbf{p} \)

Note sometimes I say \( \mathbf{r} \cdot \mathbf{\dot{p}} = \mathbf{r} \times \mathbf{\dot{p}} \)

\[ \mathbf{L} = \mathbf{r} \times \mathbf{\dot{p}} = \mathbf{\tau} \] (torque)

Energy and Work

Consider a static force field \( \mathbf{F}(\mathbf{r}) \)

Work to move a test particle from 1 to 2 is

\[ W_{1\rightarrow2} = \int_{1}^{2} \mathbf{F} \cdot d\mathbf{s} \]

along path \( s \), \( \mathbf{F} = m \frac{d\mathbf{v}}{dt} \)

\[ W_{1\rightarrow2} = \int_{1}^{2} ds \cdot m \frac{dv}{dt} = \int_{1}^{2} dv \cdot m \frac{dv}{dt} \]

\[ = m \left[ v_2 - v_1 \right] = \frac{1}{2} m (v_2^2 - v_1^2) \]

independent of the path. If \( T = \frac{1}{2} m v^2 \)

denotes the kinetic energy. The work done is

\[ W = T - U \]

If \( F \) has the special form

\[ \mathbf{F} = -\nabla U(\mathbf{r}) \]
where $U$ is the potential. Such forces are conservative, although common they are quite restrictive.

\[ W_{2} - W_{1} = T_{2} - T_{1} = -\int d\mathbf{s} \cdot \nabla U = -\int dU \]
\[ = U_{1} - U_{2} \]

So for conservative forces

\[ T_{1} + U_{1} = T_{2} + U_{2} \]

or $E = T + V$ is conserved for conservative forces.

Note two equivalent def. of conservative forces:

\[ \nabla \cdot \mathbf{F} = 0 \quad \text{for all } x \]

\[ \oint d\mathbf{s} \cdot \mathbf{F} = 0 \quad \text{for all closed paths} \]

[so work is indep. of path]