Lec. 26 Review Cont. 4/28/11

Review

Review in Mid August for Qualifier

Some definition / short answer questions but probably less than in P506

Short simplified homework problems

Core material so far in P507

Note lectures 1-25 on course web site

http://cecelia.physics.indiana.edu/p507

lec 26 will be posted Friday.

Chap. 7 Plane Waves and wave propogation

Maxwel eqs in absence of sources

$$\nabla \cdot B = 0 \qquad \nabla \times E + \partial B/\partial t = 0$$
$$\nabla \cdot D = 0 \qquad \nabla \times H - \partial D/\partial t = 0$$

assume $e^{-i\omega t}$ time dep. and

$$D = \epsilon E \quad , \quad B = \mu H$$

$$\nabla \times E - i\omega B = 0 \qquad \nabla \times B + i\omega\mu\epsilon E = 0$$

$\Rightarrow$ Wave eqs.

$$[\nabla^2 + \mu\epsilon\omega^2] E = 0$$
$$[\nabla^2 + \mu\epsilon\omega^2] B = 0$$

$k = \sqrt{\mu\epsilon}\,\omega$ wave vector

$v = \frac{\omega}{k}$ phase velocity

$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n}$, $n = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$ index of refraction

Plane waves with $E \perp B \perp \hat{n}$

Polarization and stokes parameters
Linear, circular, eliptical

Reflection / Refraction at plane interface between dielectrics

Boundary conditions

Normal comp. of D, B are cont.

Tangential comp. of E, H are cont.

Snell's law $\frac{\sin i}{\sin r} = \frac{n'}{n}$

Dispersion freq. dep. of n and ϵ

Model for dielectric $\epsilon(\omega)$ $\boxed{\text{Section 7,5}}$

Collection of harmonic osc. with freq. ω_i damping γ_i and strength f_i

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{N e^2}{\epsilon_0 m} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\omega\gamma_j}$$

N molecules per unit volume, Sum rule $\sum_j f_j = Z$

away from a resonance $\omega \neq \omega_j$

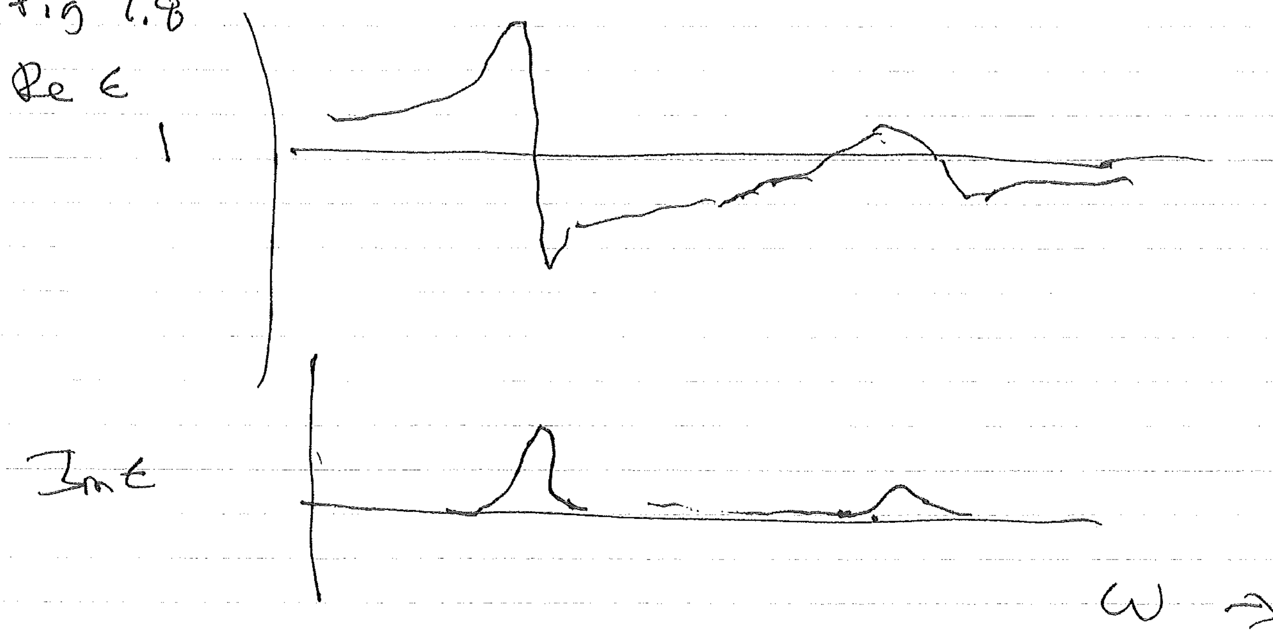
<u>Normal dispersion</u>

$\epsilon(\omega)$ increases with increasing ω

<u>Anomalous dispersion</u>

Re $\epsilon(\omega)$ has rapid ω dep.
near a resonance $\omega \approx \omega_j$ where
Im $\epsilon(\omega)$ is large

Fig 7.8



Low freq. dep. and electrical cond.

High freq. dep. and plasma freq
for $\omega \gg$ all ω_j

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

Plasma freq. ω_p

$$\omega_p^2 = \frac{N Z e^2}{\epsilon_0 m}$$

Depends on charge density and mass of charged particles (electrons).

Index of refraction and absorption coeff. of liquid water

Review ~~Fig 7.9~~

Dependence of absorption on frequency

Absorp. is small only for narrow range of visible freq. ~~0.5~~ absorption at higher and lower freq.

$n \sim 11$ low freq.
$\quad 1.34$ visible

<u>Magnetohydrodynamics</u>

In a conducting fluid or ionized gas collisions are so rapid that Ohm's law will hold

$$\vec{J} = \sigma(E + v \wedge B)$$

Where Ohm's law is generalized to include bulk motion v of fluid

Hydrodynamic eq.

cont. eq. $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$

Newton's 2nd law

$$\frac{d}{dt} p = F$$

$$\rho \frac{\partial v}{\partial t} + \rho (v \cdot \nabla) v = -\nabla p - \frac{1}{\mu}(B \wedge (\nabla \wedge B))$$

Magnetic force density $= \vec{j} \wedge \vec{B}$
with $\vec{E} = 0$ inside conductor

Alfven waves can propagate along B with velocity

$$v_A = \frac{B_0}{\sqrt{\mu \rho_0}}$$

ρ_0 = equlibrium density

7.8 Superposition of waves and group velocity

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k)\, e^{ikx - i\omega(k)t}\, dk$$

$$v_g = \frac{\partial \omega}{\partial k} = \frac{c}{n(\omega) + \omega \frac{\partial n}{\partial \omega}}$$

Spreading of a pulse as it moves in a dispersive medium.

Kramers - Kronig Relations

Causality

$\frac{\epsilon(\omega)}{\epsilon_0}$ is analytic in upper half plane

$D(x,\omega) = \epsilon(\omega)\, E(x,\omega)$

F.T. $D(x,t)$ can only depend on $E(x,t')$ for $t' \leq t$

$$\epsilon(-\omega)/\epsilon_0 = \epsilon^*(\omega^*)/\epsilon_0$$

for real ω

$$\mathrm{Im}\,\epsilon(-\omega) = -\mathrm{Im}\,\epsilon(\omega)$$

$$\mathrm{Re}\,\epsilon(-\omega) = \mathrm{Re}\,\epsilon(\omega)$$

Dispersion relation

$$\mathrm{Re}\,\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{2}{\pi} P\int_0^\infty \frac{\omega'\,\mathrm{Im}\,\epsilon(\omega')/\epsilon_0}{\omega'^2 - \omega^2}\,d\omega'$$

$$\mathrm{Im}\,\frac{\epsilon(\omega)}{\epsilon_0} = -\frac{2\omega}{\pi} P\int_0^\infty \frac{\mathrm{Re}\,\epsilon(\omega')/\epsilon_0 - 1}{\omega'^2 - \omega^2}\,d\omega'$$

Similar relations in many areas of physics.

Chap. 9

Radiating Systems

Use retarded Green's func.

$$\lim_{kr \to \infty} A(\vec{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int J(\vec{x}')\, e^{-ik\, n \cdot x'}\, d^3x'$$

Electric dipole fields

$$H = \frac{ck^2}{4\pi}\, n \wedge p\, \frac{e^{ikr}}{r}$$

$$E = Z_0\, H \wedge n$$

$$\vec{p} = \int x' \rho(x')\, d^3x' \quad \text{elec. dipole moment}$$

similar formula for magnetic dipole fields

Power radiated

$$\frac{dP}{d\Omega} = \frac{1}{2} \mathrm{Re}\left[r^2\, \hat{n} \cdot E \wedge H^* \right]$$

time average of harmonic time dep. $\langle [\mathrm{Re}\, e^{-i\omega t}]^2 \rangle$

Spherical Wave solutions to Scalar Wave Eq.

$$(\nabla^2 + k^2)\, \psi(x, \omega) = 0 \quad . \quad k = \omega/c$$

$$\psi = \sum_{lm} f_{lm}(r)\, Y_{lm}$$

$$f_{lm} \propto j_l(kr) \text{ or } n_l(kr)$$

Spherical Bessel func. $j_l \pm i\, n_l = h_l^{(1,2)}$

$$\frac{e^{ik|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|} = ik \sum_l j_l(kr_<)\, h_l^{(1)}(kr_>) \sum_m Y^*_{lm} Y_{lm}$$

$$L = \frac{1}{i}\, r \wedge \nabla$$

$$X_{lm} = \frac{L\, Y_{lm}}{\sqrt{l(l+1)}}$$

$$H = \sum_{lm} \left[a_E(lm)\, f_l\, X_{lm} - \frac{i}{k}\, a_M(l,m)\, \nabla \wedge g_l(kr) X_{lm} \right]$$

E related to curl H

Expansion of general solution of maxwell's eqs in source free region

In $r \to \infty$ limit $f_l,\ g_l \to h_l^{(1)} = (-i)^{l+1} \frac{e^{ikr}}{kr}$

$$H \to \frac{e^{ikr - i\omega t}}{kr} \sum_{lm} (-i)^{l+1} \left[a_E(lm) X_{lm} + a_M(lm)\, n \wedge X_{lm} \right]$$

$$E \to Z_0\, H \wedge n$$

Sources of multipole radiation; multipole moments

$$a_E = \frac{k^2}{i\sqrt{l(l+1)}} \int Y^*_{lm} \left\{ c\rho \frac{\partial}{\partial r}(r j_l) + ik\, \vec{r}\cdot\vec{J}\, j_l - ik \nabla\cdot(\vec{r} \wedge \mathcal{M})\, j_l \right\} d^3x$$

Similar exp. for a_M

Power radiated, angular dist. in terms of a_E, a_M

Scattering in long wavelength limit

Power radiated by $\vec{p}$, $\vec{m}$ induced electric or magnetic dipole moments

$$H = \frac{i\omega}{4\pi}(n \wedge p)\frac{1}{r^3}$$

$$E = \frac{1}{4\pi\epsilon_0}(3n\, n\cdot p - p)\frac{1}{r^3}$$

$$\frac{dP}{d\Omega} = \frac{1}{2}\operatorname{Re}\left[r^2\, n\cdot E \wedge H^*\right]$$

$$= \frac{c^2 Z_0}{32\pi^2} k^4 \left|\vec{n}\wedge\vec{p}\wedge\vec{n}\right|^2 \qquad Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

Consider incident plane wave

$$E_{inc} = \epsilon_0 E_0\, e^{ik\, n_0\cdot x}$$

$$H_{inc} = n_0 \wedge E_{inc} / Z_0$$

This will induce $\vec{p}$, $\vec{m}$ and they will produce

$$E_{sc} = \frac{1}{4\pi\epsilon_0} k^2 \frac{e^{ikr}}{r}\left(n\wedge p\wedge n - n\wedge m/c^2\right)$$

$$H_{sc} = n \wedge E_{sc} / Z_0$$

$$\frac{d\sigma}{d\Omega}(n\epsilon;\, n_0\epsilon_0) = \frac{r^2 \frac{1}{2Z_0}\left|\epsilon^*\cdot E_{sc}\right|^2}{\frac{1}{2Z_0}\left|\epsilon_0^*\cdot E_{inc}\right|^2}$$

$$= \frac{k^4}{(4\pi\epsilon_0 E_0)^2}\left|\epsilon^*\cdot p + (n\wedge\epsilon^*)\cdot\frac{m}{c^2}\right|^2$$

Example scattering by small dielectric sphere

$$\vec{p} = 4\pi \epsilon_0 \left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right) a^3 \vec{E}_{inc}$$

In general $\vec{p} = \lambda \vec{E}_{inc}$, $\vec{m} = 0$

$$\frac{d\sigma}{d\Omega} = k^4 a^6 \left|\frac{\epsilon_r - 1}{\epsilon_r + 2}\right|^2 |\epsilon^* \cdot \epsilon_0|^2$$

Unpolarized cross section

(1) Sum over final polarization

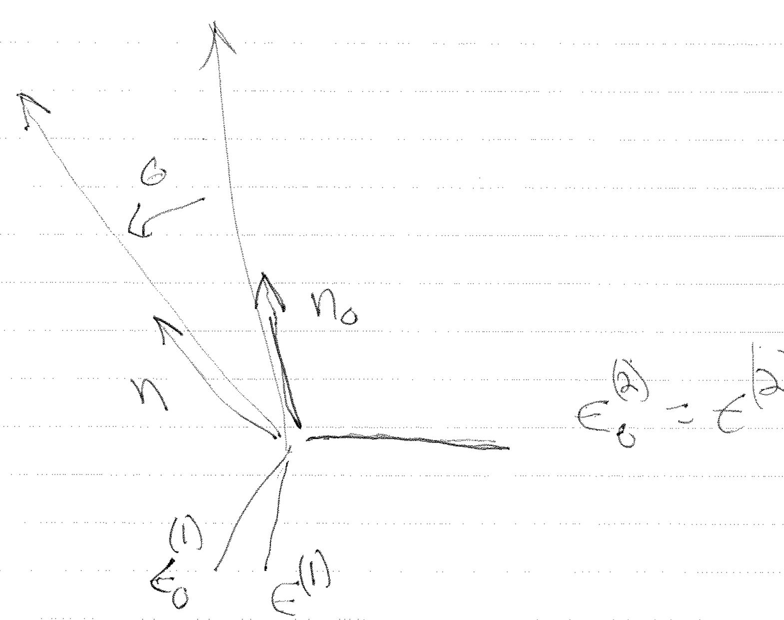
(2) Average over initial polarization

$$\overline{\frac{d\sigma}{d\Omega}} = k^4 a^6 \left|\frac{\epsilon_r - 1}{\epsilon_r + 2}\right|^2 \frac{1}{2}\sum_{j=1}^{2}\sum_{i=1}^{2} |\epsilon^{*(j)} \cdot \epsilon_0^{(i)}|^2$$

$\epsilon^{(2)*} \cdot \epsilon_0^{(2)} = 1$

$\epsilon^{(1)*} \cdot \epsilon_0^{(1)} = \cos\theta$

$\epsilon^{(j)*} \cdot \epsilon_0^{(i)} = 0$ $i \neq j$



$$\frac{d\sigma}{d\Omega} = k^4 a^6 \left(\frac{\epsilon_r - 1}{\epsilon_r + 2}\right)^2 \frac{1}{2}\left(1 + \cos^2\theta\right)$$

Review Perturbation theory and Rayleigh scattering

$$\frac{d\sigma}{d\Omega} \propto \omega^4$$

Why sky is ~~bote~~ blue

Chap. 11 Relativity

Postulates

1. Laws of nature have same form in any inertial frame
2. Speed of light is c in any inertial frame.

4 Vectors transform $A_\mu \to A'_\mu$

$$A'_0 = \gamma\left(A_0 - \vec{\beta}\cdot\vec{A}\right)$$

$$A'_\parallel = \gamma\left(A_\parallel - \beta A_0\right)$$

$$A'_\perp = A_\perp$$

$A \cdot B = A_0 B_0 - \vec{A}\cdot\vec{B}$ is invariant

Proper time $c^2(d\tau)^2 = c^2(dt)^2 - (d\vec{x})^2$

$$d\tau = dt\,(1-\beta^2)^{1/2} \qquad \vec{\beta} = \frac{1}{c}\frac{d\vec{x}}{dt}$$

Time in lab frame

$$t_1 - t_2 = (\tau_1 - \tau_2)\,\gamma$$

Addition of velocities

$$U_{\parallel} = \frac{U'_{\parallel} + V}{1 + \vec{V}\cdot\vec{U}'/c^2}$$

$$U_{\perp} = U'_{\perp} \,/\, \gamma_v \left[1 + (\vec{V}\cdot\vec{U}')/c^2\right]$$

Four velocity $\cancel{U_0 = \frac{\partial x_0}{\partial \tau} = \gamma c}$

$$U_\mu = (\gamma c, \gamma \vec{v}) \qquad \cancel{U_i = \gamma \vec{v}}$$

$$= \frac{\partial x_\mu}{\partial \tau}$$

Momentum $\vec{p} = \gamma m \vec{u}$

$$E = \gamma m c^2$$

Tensors of rank k

Contravariant vector A^α

$$A'^\alpha = \frac{\partial x'^\alpha}{\partial x^\beta} A^\beta$$

Covariant vector

$$B'_\alpha = \frac{\partial x^\beta}{\partial x'^\alpha} B_\beta$$

Metric tensor

$$g_{\alpha\beta} = \begin{bmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{bmatrix}$$

$$A_\alpha = g_{\alpha\beta} A^\beta$$

$$A^\alpha = g^{\alpha\beta} A_\beta$$

$g_{\alpha\beta}$ converts contravariant vec. to covariant vector.

$$A^\alpha = (A^0, A^1, A^2, A^3)$$

$$A_\alpha = (A^0, -A^1, -A^2, -A^3)$$

Matrix rep. of Lorentz trans.

$$X' = A\,x \qquad x = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$A_{\text{boost}}(\vec{\beta}) = \begin{bmatrix} \gamma & -\gamma\beta_1 & -\gamma\beta_2 & -\gamma\beta_3 \\ -\gamma\beta_1 & 1+\lambda\beta_1^2 & \lambda\beta_1\beta_2 & \lambda\beta_1\beta_3 \\ -\gamma\beta_2 & \lambda\beta_1\beta_2 & 1+\lambda\beta_2^2 & \lambda\beta_2\beta_3 \\ -\gamma\beta_3 & \lambda\beta_1\beta_3 & \lambda\beta_2\beta_3 & 1+\lambda\beta_3^2 \end{bmatrix}$$

$$\lambda = \frac{\gamma - 1}{\beta^2}$$

Covariance of E+M

$$\frac{dp}{dt} = q\left(\vec{E} + \frac{\vec{V}}{c} \wedge \vec{B}\right)$$

Write as

$$\frac{d\vec{p}}{d\tau} = \frac{q}{c}\left(U_0 \vec{E} + \vec{U} \wedge \vec{B}\right)$$

and $\frac{dp_0}{d\tau} = \frac{q}{c} \vec{U} \cdot \vec{E}$

$\vec{E}$, $\vec{B}$ must transform so above is invariant.

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha$$

$$= \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix}$$

Field strength tensor

$$\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c} J^\beta$$

covariant form of Maxwell eqs.

$\Rightarrow$ $E_x' = E_x$ consider boost in x direction

$E_y' = \gamma(E_y - \beta_1 B_z)$

$E_z' = \gamma(E_z + \beta_1 B_y)$

E and B mix under Lorentz trans.

Relativistic eq. for spin

Motion in E + M fields

Example crossed E and B fields ~~as~~ can act as velocity filter.

Lagrangian for E + M field

$$\mathcal{L} = -\frac{1}{16\pi} F_{\alpha\beta} F^{\alpha\beta} - \frac{1}{c} A_\alpha J^\alpha$$

$$\partial^\beta \frac{\partial \mathcal{L}}{\partial \partial^\beta A^\alpha} - \frac{\partial \mathcal{L}}{\partial A^\alpha} = 0 \Rightarrow \text{Maxwell eqs.}$$