4/14/11

Lec. 22 g-2 of Muon

Last time

Defined spin 4 vector

S^α with $U_\alpha S^\alpha = 0$

so that time component of S^α is zero in particles rest frame

Found relativistic generalization of

$$\frac{d\vec{s}}{dt'} = \frac{ge}{2mc}\,\vec{s} \wedge \vec{B}' \qquad \boxed{\vec{\mu} = \frac{ge}{2mc}\,\vec{s}}$$

as Magnetic moment.

$$\frac{dS^\alpha}{d\tau} = \frac{e}{mc}\left[\frac{g}{2} F^{\alpha\beta} S_\beta + \frac{1}{c^2}\left(\frac{g}{2}-1\right) U^\alpha S_\lambda F^{\lambda\mu} U_\mu\right]$$

BMT eq.

We used this to find $\frac{d\vec{s}}{dt}$ valid for any E, B in particle's rest frame.

Also

$$\frac{d\vec{\beta}}{dt} = \frac{e}{\gamma mc}\left[\vec{E} + \vec{\beta}\wedge\vec{B} - \vec{\beta}(\vec{\beta}\cdot\vec{E})\right]$$

For Lorentz force law

Torque in rest frame found by boosting B, E

$$\frac{1}{\gamma}\vec{F}' = \frac{ge}{2mc}\vec{S}\wedge\left[\vec{B} - \left(\frac{\gamma}{\gamma+1}\right)\beta\cdot\vec{B}\,\beta - \beta\times\vec{E}\right]$$

This gives Thomas's eq.

$$\frac{d\vec{S}}{dt} = \frac{e}{mc}\vec{S}\wedge\left[\left(\frac{g}{2}-1+\frac{1}{\gamma}\right)\vec{B} - \left(\frac{g}{2}-1\right)\frac{\gamma}{\gamma+1}\,\beta\cdot\vec{B}\,\beta - \left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\vec{\beta}\wedge\vec{E}\right]$$

$\boxed{\text{end } 4/12/11}$

Rate of change of longitudinal polarization

$$\frac{d}{dt}(\hat{\beta}\cdot\vec{S}) = \hat{\beta}\cdot\frac{d\vec{S}}{dt} + \frac{1}{\beta}(\vec{S} - \hat{\beta}\cdot\vec{S}\hat{\beta})\cdot\frac{d\vec{\beta}}{dt}$$

$$= -\frac{e}{mc}\vec{S}_\perp\cdot\left[\left(\frac{g}{2}-1\right)\hat{\beta}\wedge\vec{B} + \left(\frac{g\beta}{2}-\frac{1}{\beta}\right)\vec{E}\right]$$

$\vec{S}_\perp$ = component of $\vec{S}$ $\perp$ to velocity

Define anomaly $a \equiv \left(\frac{g-2}{2}\right)$

$$\frac{d}{dt}(\hat{\beta}\cdot\vec{S}) = -\frac{e}{mc}\vec{S}_\perp\cdot\left[a\,\hat{\beta}\wedge\vec{B} + \left(a+1-\frac{1}{\beta^2}\right)\beta\vec{E}\right]$$

$$a + 1 - \frac{1}{\beta^2} = a - \frac{1}{\gamma^2 - 1}$$

$$a \simeq \alpha/2\pi$$

$$\frac{d}{dt}(\hat{\beta}\cdot\vec{s}) = \frac{-e}{mc}\,\vec{s}_\perp\left[\cdot\; a\,\hat{\beta}\wedge\vec{B} + \left(a - \frac{1}{\gamma^2-1}\right)\beta\vec{E}\right]$$

a particle in ~~uniform~~ E and B fields will have its spin presses at a rate $\propto$ a compared to its momentum.

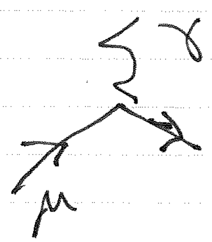
For a magic momentum where

$$a = \frac{1}{\gamma^2 - 1}$$

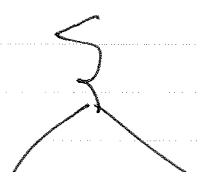
The spin preassion is insensitive to the electric field.

Example anomalous magnetic moment of the muon

See Z. Zhang "Muon g-2 a mini review", arXiv: 0801.4905



lowest order coupling of photon to muon
$g=2$, $a=0$
from Dirac eq.



$$a = \frac{\alpha}{2\pi}$$ Schwinger 1948

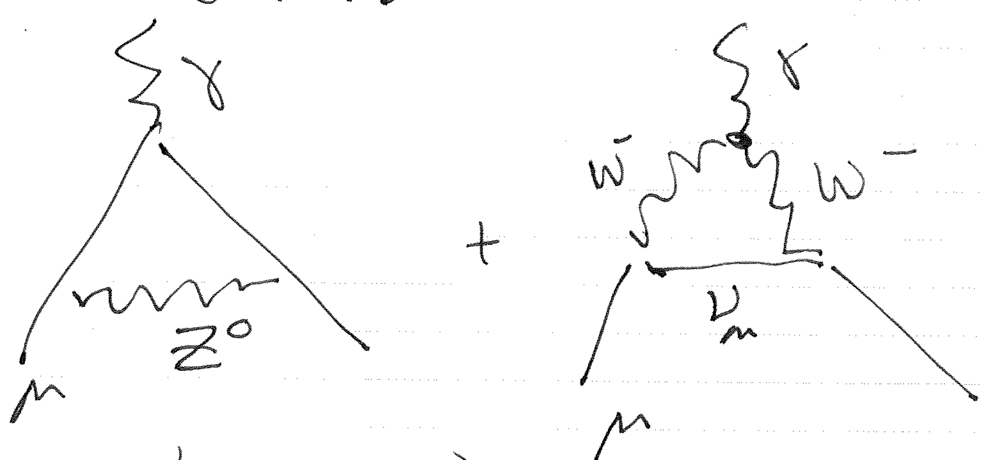
+ 

example of higher order (vacuum polarization) correction to a

$$a_\mu^{QED} = \frac{\alpha}{2\pi} + (0.765\ldots)\left(\frac{\alpha}{\pi}\right)^2 + (24.05\ldots)\left(\frac{\alpha}{\pi}\right)^3 + (130.99\ldots)\left(\frac{\alpha}{\pi}\right)^4 + [663 \pm 20]\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

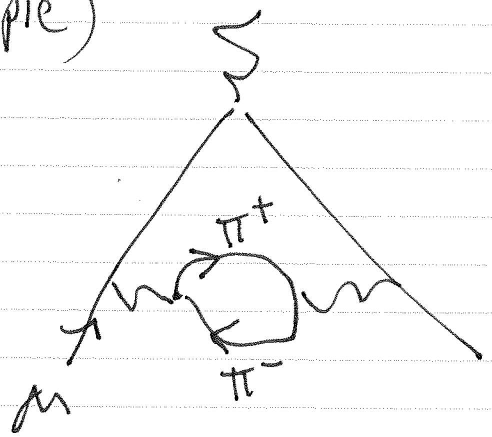
It has now been calculated through α^5

Also weak contribution



Weak

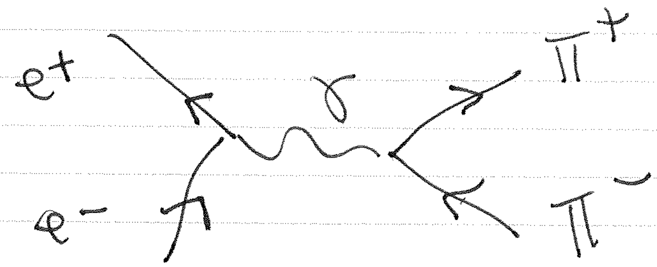
also have hadronic corrections
(for example)



These are calculated from measured

$e^+e^- \rightarrow$ Virtual $\gamma \rightarrow$ Hadrons

data



$a_\mu^{Had} \simeq 6.91 \times 10^{-8}$

~~The BNL~~

$a_\mu = a_\mu^{QED} + a_\mu^{Weak} + a_\mu^{Had} + a_\mu^{New\ physics}$

a_μ (Measured BNL 2004)

$= 0.0011659208 0 \pm 63$ (0.54 ppm)

protons $\rightarrow$ make $\pi^\pm$ ~~der~~ that decay with very uniform magnetic field and electrostatik focussing.

Muons are at magic momentum

$$a \doteq \frac{\alpha}{2\pi} = \frac{1}{\gamma^2-1} \doteq 0.00116$$

$$\alpha = \frac{1}{137.036}$$

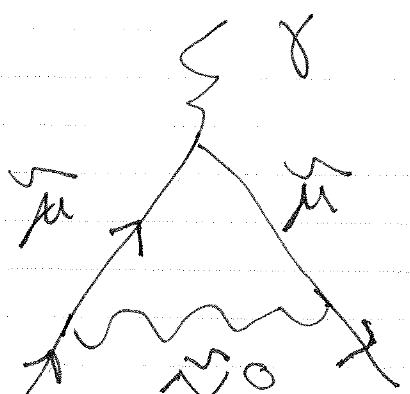
$$\gamma \simeq 29.36$$

$$E_{\mu} = \gamma m_{\mu} c^2 \qquad m_{\mu} = 105.7 \ \mathrm{MeV}/c^2$$

$$= 3.1 \ \mathrm{GeV}$$

The theoretical prediction for a_{μ} differs from the observed value by about $3.3\,\sigma$.

This could be the first sign of new physics. For example new supersymmetric particles can contribute to a_{μ}



Many checks of hadronic contribution now underway.

Improve accuracy of BNL experiment perhaps at Fermi Lab.

Chapter 12

Dynamics of Relativistic Particles and E+M fields.

We have talked about Kinematics in Chap. 11 For example how E, B transform under L.T.

Now let us talk about dynamics.

Consider particle in external E+M fields

$$\frac{d\vec{p}}{dt} = e\left[\vec{E} + \frac{\vec{v}}{c}\wedge\vec{B}\right]$$

$$\frac{dE}{dt} = e\,\vec{v}\cdot\vec{E}$$

Lorentz covariant version

$$\frac{dU^\alpha}{d\tau} = \frac{e}{mc}F^{\alpha\beta}U_\beta$$

with $U^\alpha = (\gamma c, \gamma\vec{v}) = p^\alpha/m$ = 4 vel.

Derive this from Lagrangian dynamics

Minimize action

Euler - Lagrange eqs

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

Rewrite action in terms of proper time

$$A = \int_{\tau_1}^{\tau_2} \gamma L \, d\tau$$

Action is invariant, as is τ

$\Rightarrow$ γL is Lorentz invariant.

Lagrangian for a free particle can be a function of its velocity and mass. It can not depend on position. Only Lorentz invariant function of velocity is $U_\alpha U^\alpha = c^2$

$$L \propto \gamma^{-1} = \sqrt{1-\beta^2}$$

$$L_{free} = -mc^2\left(1 - \frac{u^2}{c^2}\right)^{1/2}$$

$$\frac{\partial L_{free}}{\partial u} = -\frac{1}{2} mc^2\left(1-\frac{u^2}{c^2}\right)^{-1/2} \frac{-2u}{c^2}$$

$$= \gamma m \vec{u}$$

Therefore

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = \frac{d}{dt}\left(\gamma m u_i\right) = 0$$

Now consider electric field

$L_{nr} = T - V, \qquad V = e\Phi$

$L_{int} \rightarrow L^{NR'}_{int} = -e\Phi$

Try

$$L_{int} = -\frac{e}{\gamma c} U_\alpha A^\alpha$$

For a slowly moving particle this reduces to $-e\Phi$

$$\boxed{L = -mc^2\sqrt{1-\frac{u^2}{c^2}} + \frac{e}{c}\vec{u}\cdot\vec{A} - e\Phi}$$

From this the lagrang eq. of motion $\Rightarrow$ Lorentz force Law.
see 9a

Hamiltonian

Conjugate momentum

$$P_i \equiv \frac{\partial L}{\partial u_i} = \gamma m u_i + \frac{e}{c}A_i$$

$$\vec{P} = \vec{p} + \frac{e}{c}\vec{A}$$

$$H = \sum_i P_i u_i - L$$

$$\vec{u} = \frac{\vec{P} - \frac{e}{c}\vec{A}}{\gamma m}$$

$$\cancel{H = \frac{P^2 - \frac{e}{c}\vec{P}\cdot\vec{A}}{\gamma m} - L}$$

Derivation of Lorentz Force Law

$$\frac{\partial L}{\partial v_i} = \gamma m v_i + \frac{e}{c} A_i$$

$$\frac{\partial L}{\partial x_i} = \frac{e}{c} \vec{v} \cdot \frac{\partial \vec{A}}{\partial x_i} - e \frac{\partial \Phi}{\partial x_i}$$

$$\frac{d}{dt} \frac{\partial L}{\partial v_i} = \frac{d}{dt}\left(\gamma m v_i\right) + \frac{e}{c} \frac{d}{dt}(A_i)$$

$$A_i = A_i(\vec{x}, t)$$

$$\frac{d}{dt} A_i = \frac{\partial A_i}{\partial t} + \frac{\partial A_i}{\partial x_j} \frac{\partial x_j}{\partial t} = \frac{\partial A_i}{\partial t} + \vec{v} \cdot \vec{\nabla} A_i$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_i}\right) = \frac{d}{dt}\left(\gamma m v_i\right) + \frac{e}{c} \frac{\partial A_i}{\partial t} + \frac{e}{c} \vec{v} \cdot \vec{\nabla} A_i$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_i}\right) - \frac{\partial L}{\partial x_i} = \frac{d}{dt}\left(\gamma m v_i\right) + \frac{e}{c} \frac{\partial A_i}{\partial t} + \frac{e}{c} \vec{v} \cdot \vec{\nabla} A_i$$

$$+ e \frac{\partial}{\partial x_i} \Phi - \frac{e}{c} \vec{v} \cdot \frac{\partial}{\partial x_i} \vec{A}$$

Now $\vec{E} = -\nabla \Phi - \frac{1}{c} \frac{\partial A}{\partial t}$

$$\frac{d}{dt} p_i = e E_i + \frac{e}{c}\left[\vec{v} \cdot \frac{\partial}{\partial x_i} \vec{A} - \vec{v} \cdot \vec{\nabla} A_i\right]$$

$$\frac{d}{dt} \vec{p} = e \vec{E} + \frac{e}{c} \vec{v} \wedge \vec{B}$$

Lorentz force law

$$m^2 \frac{v^2}{1-\frac{v^2}{c^2}} = \left(\vec{P} - \frac{e}{c}\vec{A}\right)^2$$

$$\boxed{v = \frac{cP - eA}{\sqrt{\left(P - \frac{eA}{c}\right)^2 + m^2c^2}}}$$

$$1 - \frac{v^2}{c^2} = 1 - \frac{\left(P - \frac{e}{c}A\right)^2}{\left(P - \frac{e}{c}A\right)^2 + m^2c^2} = \frac{m^2c^2}{\left(P - \frac{e}{c}A\right)^2 + m^2c^2}$$

$$H = \vec{P}\cdot\vec{v} - L$$

$$= P\cdot v + \frac{mc^2}{\gamma} - \frac{e}{c}\vec{v}\cdot\vec{A} + e\Phi$$

$$= \frac{cP\cdot\left(P - \frac{e}{c}A\right)}{\left[\left(P - \frac{e}{c}A\right)^2 + m^2c^2\right]^{1/2}} + \frac{mc^2(mc)}{\left[\left(P - \frac{e}{c}A\right)^2 + m^2c^2\right]^{1/2}}$$

$$- \frac{e\left(P - \frac{e}{c}A\right)\cdot A}{\sqrt{\left(P - \frac{e}{c}A\right)^2 + m^2c^2}} + e\Phi$$

$$= c\left\{\frac{\left(P - \frac{e}{c}A\right)^2 + (mc)^2}{\sqrt{\left(P - \frac{e}{c}A\right)^2 + m^2c^2}}\right\} + e\Phi$$

$$\boxed{H = \left[\left(P - \frac{e}{c}A\right)^2 + m^2c^2\right]^{1/2} c + e\Phi}$$