P507 Lec. 1 1/11/11

~~\$~~ P507 is continuation of P506
We will cover parts of chapters 7, 9, 10, 11
and 12

Final exam from P506
Total 108
ave 77
High 108

Read Chap. 7

Plane waves in nonconducting medium.
Consider Maxwell Eqs in source free
region $\rho = \vec{J} = 0$

$$\nabla \cdot B = 0 \qquad \nabla \times E + \frac{\partial B}{\partial t} = 0$$
$$\nabla \cdot D = 0 \qquad \nabla \times H - \frac{\partial D}{\partial t} = 0$$

Assume harmonic time dep. $e^{-i\omega t}$ and
represent arbitrary solution by Fourier superposition

Equations for amplitudes $E(\omega, \vec{x})$ etc

$$\nabla \cdot B = \nabla \cdot D = 0 \qquad \nabla \times E - i\omega B = 0$$
$$\nabla \times H + i\omega D = 0$$

For uniform isotropic linear medium

$D = \epsilon E$, $B = \mu H$

$\nabla \times E - i\omega B = 0$, $\nabla \times B + i\omega\mu\epsilon E = 0$

take curl

$$\nabla \times \nabla \times E = i\omega \nabla \times B = \omega^2 \mu\epsilon E$$

$$-\nabla^2 E - \omega^2 \mu\epsilon E = 0 \qquad \text{given } \nabla \cdot E = 0$$

Wave eqs

$$\left[\nabla^2 + \mu\epsilon\omega^2\right]\vec{E} = 0$$
$$\left[\nabla^2 + \mu\epsilon\omega^2\right]\vec{B} = 0$$

Look for plane wave solution $e^{i\vec{k}\cdot\vec{x} - i\omega t}$

$$k = \sqrt{\mu\epsilon}\,\omega$$

Phase velocity of wave

$$v \equiv \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n} , \qquad n = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$$

$$c = \sqrt{\frac{1}{\mu_0\epsilon_0}}$$

Index of refraction n

Note $n = n(\omega)$ usually

$$U_k(x,t) = a\,e^{ik(x - vt)} + b\,e^{-ik(x+vt)}$$

If medium is nondispersive [n indep. of ω]

then fourier superposition in k

$$U(x,t) = f(x - vt) + g(x+vt)$$

These waves propagate without changing shape. However can't write general solution this way if $n = n(\omega)$ because then v depends on k.

Consider E+M plane wave of frequency ω and wave vector $\vec{k} = k\vec{n}$

Note $|\vec{n}|$ is not index of refraction!
$|\vec{n}| = 1$ unit vector

$$E(x,t) = \mathcal{E}\, e^{ik\, n\cdot x - i\omega t}$$
$$B(x,t) = \mathcal{B}\, e^{ik\, n\cdot x - i\omega t}$$

$\mathcal{E}, \mathcal{B}$ are constant vectors

$$k^2 n\cdot n = \mu\epsilon\omega^2$$
$$n\cdot\mathcal{E} = n\cdot\mathcal{B} = 0 \qquad \nabla\cdot E = 0,\ \nabla\cdot B = 0$$

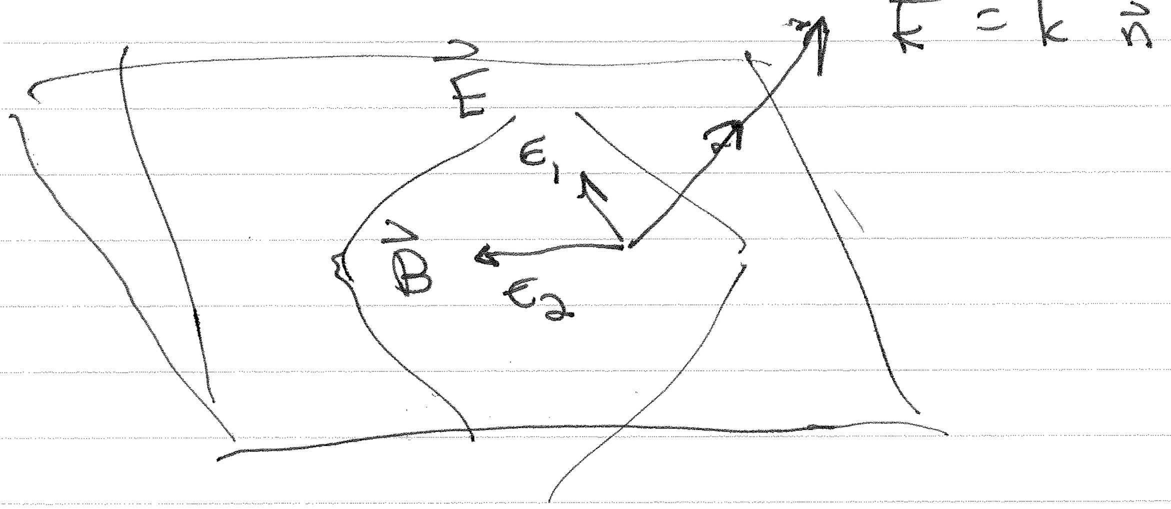
$$B = \frac{1}{i\omega}\nabla\times E \Rightarrow \qquad \mathcal{B} = \sqrt{\mu\epsilon}\,\vec{n}\times\mathcal{E}$$

If index of refraction n is real $\mathcal{B}$ and $\mathcal{E}$ have same phase
Introduce set of orthogonal unit vectors
$$\{\vec{\epsilon}_1, \vec{\epsilon}_2, \vec{n}\}$$

$$\mathcal{E} = \epsilon_1 E_0 \quad , \quad \mathcal{B} = \epsilon_2 \sqrt{\mu\epsilon}\, E_0$$

$$\text{or} \quad \mathcal{E} = \epsilon_2 E_0' \qquad \mathcal{B} = -\epsilon_1 \sqrt{\mu\epsilon}\, E_0'$$

E_0, E_0' are possibly complex constants.



Transverse plane wave propagating in $\vec{n}$ direction with two orthogonal polarization vectors ϵ_1, ϵ_2.

Note the wave vector $\vec{k}$ can be complex. In this case define $\vec{n}$ so that

$$\vec{k} = k\,\vec{n} \quad \text{and} \quad \vec{n}\cdot\vec{n} = 1$$

Note this is not

$$|\vec{n}|^2 = 1$$

$$e^{ik\,n\cdot x - i\omega t} = e^{-k\,\vec{n}_i\cdot\vec{x}}\, e^{ik\,n_R\cdot x - i\omega t}$$

The wave will have an exp. increasing or decreasing amplitude

$\Rightarrow$ Called an inhomogeneous plane wave

$$\vec{n}\cdot\vec{n} = 1$$

has real and imaginary parts

$$n_R^2 - n_I^2 = 1$$

$$\vec{n}_R \cdot \vec{n}_I = 0$$

$\vec{n}_R$ and $\vec{n}_I$ are orthogonal

$$\vec{n} = \vec{e}_1 \cosh\theta + i\,\vec{e}_2 \sinh\theta$$

e_1 and e_2 are real unit vectors ~~in~~ in x and y directions, θ is real constant so that $n_R^2 - n_I^2 = 1$

The most general vector $\mathcal{E}$ satisfying

$n \cdot \mathcal{E} = 0$ is

$$\mathcal{E} = (i\,\vec{e}_1 \sinh\theta - \vec{e}_2 \cosh\theta)A + e_3 A'$$

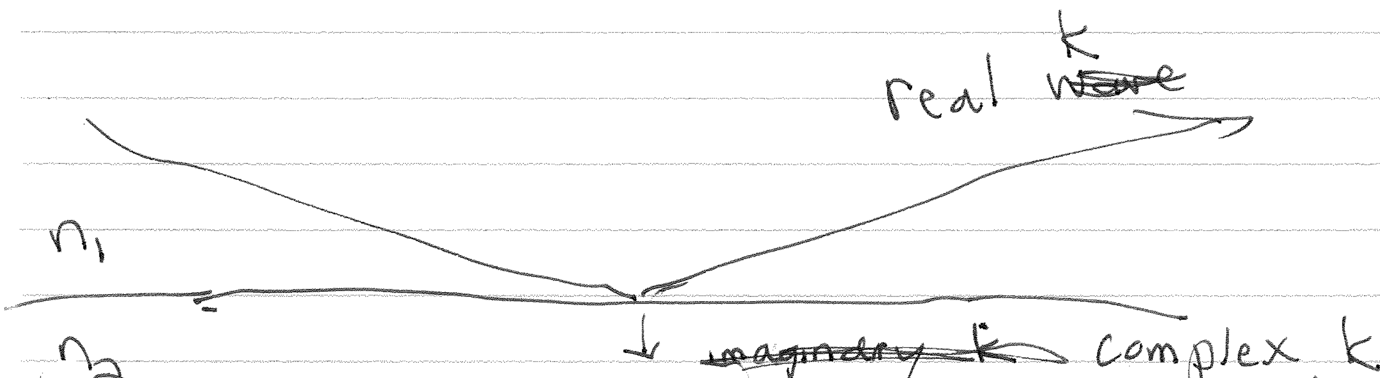
Where A, A' are complex constants.

$\vec{n}\cdot\mathcal{E} = (i\sinh\theta\cosh\theta - i\sinh\theta\cosh\theta)A = 0$ ✓

end 1/11/11

We encounter inhomogeneous plane waves in discussion of total internal reflection and refraction in a conducting medium.

Total internal reflection



so wave decays exp. and does not propagate.

Linear and Circular Polarization; Stokes parameters.

Two linear polarizations

$$\vec{E}_1 = \vec{\epsilon}_1 E_1 e^{i\vec{k}\cdot x - i\omega t}$$
$$\vec{E}_2 = \vec{\epsilon}_2 E_2 e^{ik\cdot x - i\omega t}$$

$$B_j = \sqrt{\mu\epsilon}\ \underbrace{\vec{k} \times \vec{E}_j}_{k} \qquad j = 1, 2$$

can be combined to give most general homogeneous plane wave propagating in direction $\vec{k} = k\vec{n}$