Read chap. 4 Pot. from localized charge dist.

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int d^3x'\,\rho(x')\,\frac{1}{|\vec{x}-\vec{x}'|}$$

$$= \frac{1}{\epsilon_0}\sum_{lm}\frac{1}{2l+1}\int Y^*_{lm}(\theta'\phi')\,\frac{r_<^l}{r_>^{l+1}}\,\rho(\vec{x}')\,d^3x' \quad Y_{lm}(\theta\phi)$$

If $\rho(x')$ is localized to a small volume and you are interested in the pot. outside than

$$r_< = r' \quad , \quad r_> = r$$

$$\boxed{\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left(\frac{4\pi}{2l+1}\right) q_{lm}\,\frac{Y_{lm}(\theta,\phi)}{r^{l+1}}}$$

Here the multipole moments are

$$q_{lm} \equiv \int d^3x'\,\rho(x')\,r'^l\,Y^*_{lm}(\theta',\phi')$$

$$q_{00} = \frac{1}{\sqrt{4\pi}}\int d^3x\,\rho(x) = \frac{1}{\sqrt{4\pi}}\,q$$

$$q_{11} = -\sqrt{\frac{3}{8\pi}}\int d^3x'\,\rho(x')\,(x'-iy') = -\sqrt{\frac{3}{8\pi}}\,(p_x - ip_y)$$

$$q_{10} = \sqrt{\frac{3}{4\pi}}\int \rho(x')\,d^3x'\,z' = \sqrt{\frac{3}{4\pi}}\,p_z$$

Note $q_{l\,-m} = (-1)^m\,q^*_{lm}$

Multipole moments

Spherical

$$q_{lm} \equiv \int d^3x' \, \rho(x') \, r'^l \, Y^*_{lm}(\theta'\phi')$$

Cartesian $q = \int d^3x' \, \rho(x')$ charge

$$\vec{p} = \int \vec{x}' \, \rho(x') \, d^3x$$

dipole moment

$$Q_{ij} = \int \left(3x'_i x'_j - r'^2 \delta_{ij}\right) \rho(\vec{x}') \, d^3x'$$

traceless quad. moment tensor.

$$q_{11} = -\sqrt{\frac{3}{8\pi}} \left(p_x - i p_y\right)$$

$$q_{10} = \sqrt{\frac{3}{4\pi}} \, p_z$$

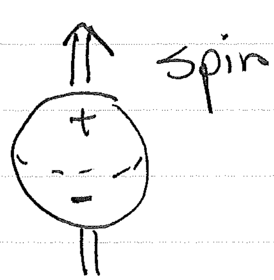
$$q_{20} = \frac{1}{2}\sqrt{\frac{5}{4\pi}} \, Q_{33} \quad \text{etc.}$$

In particle and nuclear physics electric dipole moments are much smaller ~~than~~ and much more interesting than they have any right to be.

Retroreflectors on Moon
Planet in Habital zone.

A particle be it "elementary" such as an electron / or composit such as a proton can have a magnetic dipole moment (we will talk about magnetic dipole moments in Chap. 5). The dipole moment points in the direction of the particles spin. Indeed the magnetic moment of the proton was measured in the 1930s and found to be large than expected ⇒ suggesting the proton has a more complex structure that we now know involves an extended distribution of quarks + gluons.

In contrast to magnetic moments no elementary particle or nucleus has been observed to have an electric dipole moment. Why?.

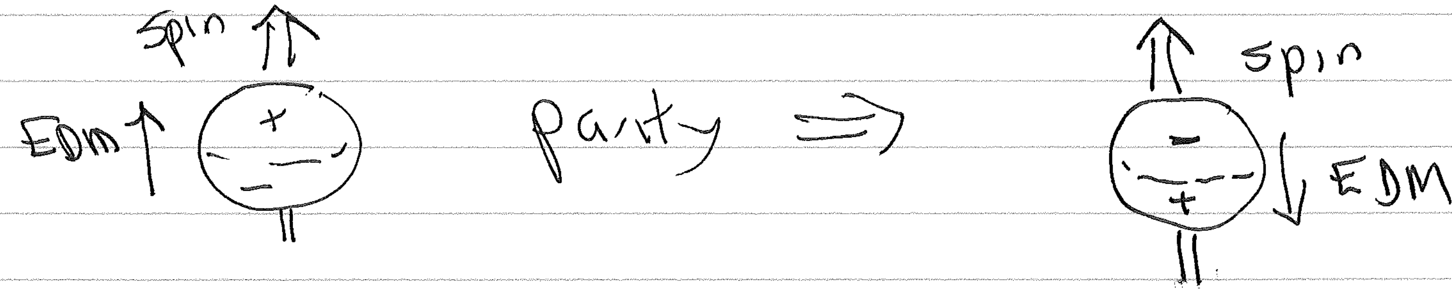


We can think of an EDM as a small separation of charge where ~~differ~~ the top half of the particle has a small + charge

However the EDM is greatly constrained by symmetry.

Parity transformation is reflection through the origin

$$\vec{x} \Rightarrow -\vec{x}$$



Under parity the spin vector is unchanged (as is the magnetic field). However the + charge originally on top becomes on the bottom so the EDM flips direction under parity.

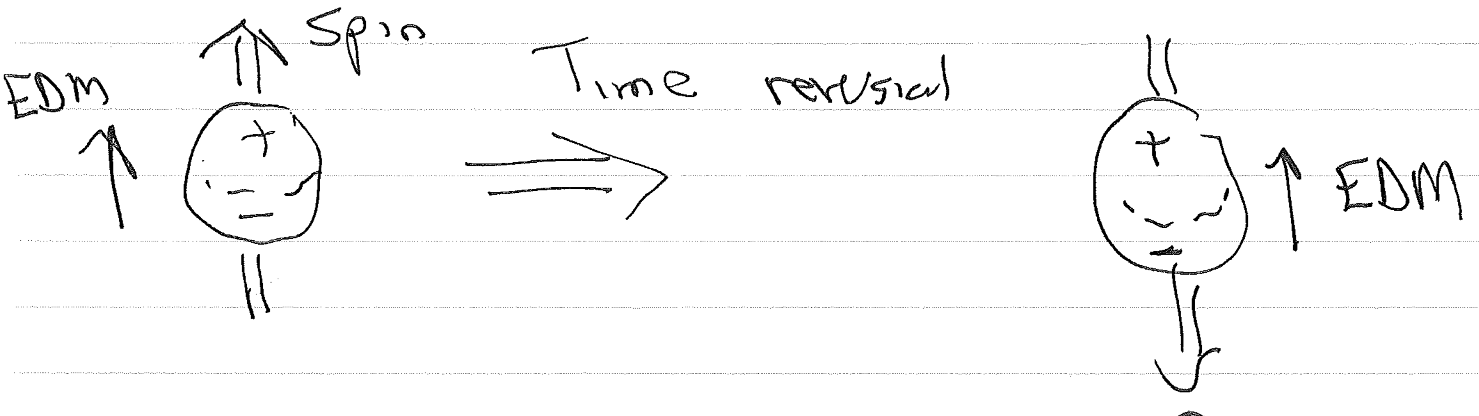
If a particle originally had an EDM alligned with its spin ⇒ than after a parity transformation the EDM ⇒ anti alligned.

EDM's are odd under parity.

If all the interactions conserve parity than the EDM $\equiv 0$.

However the weak interactions violate parity. In a proton the quarks feel weak interactions so this could give rise to a small EDM. However EDMs are also odd under time reversal.

If you play a movie backwards the particle spins in the opposite direction. However the top and bottom don't change position



So if EDM started aligned with the spin it becomes anti aligned under time reversal

EDM violate both parity and time reversal!

In standard model P is violated but T is mostly conserved.

Electric dipole moments are predicted to be very small.

Sensitive ~~mea~~ searches for EDMs ⇒ very small energy shift when particles spin is flipped in an electric field, provide sensitive probes of physics beyond the standard model.

T violation is closely related to CP violation

~~G~~ P Parity space inversion

C Charge conjugation particles ⟺ antiparticles

T time reversal.

We think under very general condition that CPT is conserved.

Therefore T violation suggests CP violation.

We think we need additional sources of CP violation beyond small amount in standard model to explain why Universe has more baryons than antibaryons.

Searching for EDMs is a good way to probe additional sources of CP violation.

E. Quadrupole Moments

$$q_{20} = \frac{1}{2}\sqrt{\frac{5}{4\pi}} \int \left(3z'^2 - r'^2\right) \rho(\vec{x}')\, d^3x'$$

$$= \frac{1}{2}\sqrt{\frac{5}{4\pi}}\, Q_{33}$$

end 10/5/10

$$q_{21} = -\frac{1}{3}\sqrt{\frac{15}{8\pi}} \left(Q_{13} - i\, Q_{23}\right)$$

$$q_{22} = \frac{1}{12}\sqrt{\frac{15}{2\pi}} \left(Q_{11} - 2i\, Q_{12} - Q_{22}\right)$$

$$Q_{ij} = \int \left(3x_i' x_j' - r'^2 \delta_{ij}\right) \rho(\vec{x}')\, d^3x'$$

No symmetries such as P and T constrain EQM. Consider deuteron bound state of a proton + neutron. Strong force between n and p is strongly spin dependent. ⇒ Deuteron is spin 1 where spins of p and n are alligned. In fact perfeed