9/4/98

Lecture #3 Probability

Last time: Time evolution from

$$i\hbar \frac{\partial \Psi}{\partial t}(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\,\Psi$$

I just tell you this gives $\Psi(x,t)$ for all $t > 0$ given V and $\Psi(x,0)$.

Prob. interp. of wave function

$$P(x \to x+dx) = \Psi^*(x,t)\,\Psi(x,t)\,dx$$

Normalization

$$\int_{-\infty}^{\infty} dx\ \Psi^*(x,t)\,\Psi(x,t) = 1$$

Prob. to find particle somewhere is 1
Chose $\Psi(x,0)$ so that

$$\int_{-\infty}^{\infty} dx\ \Psi^*(x,0)\,\Psi(x,0) = 1$$

Show later $\displaystyle \frac{d}{dt}\int_{-\infty}^{\infty} dx\ \Psi^*(x,t)\,\Psi(x,t) = 0$

General background on Probability

Monte Carlo methods very important in Physics calculations. Example Monte Carlo evaluation of an integral

$$I = \int_0^1 dx\ f(x) \simeq \frac{1}{N} \sum_{i=1}^{N} f(x_i)$$

x_i are random numbers uniformly distributed on $(0,1)$.

Some references Press et al "Numerical Recipes" discusses random number generators. S. Koonin, "Computational Physics" chap 8

Consider first a <u>Discrete Variable</u> only takes on integer values

Example room full of 14 people of age

$14, 15, \underbrace{16, 16, 16}_{3} \qquad \underbrace{22, 22}_{2} \quad \underbrace{24, 24}_{2}$

$\underbrace{25, 25, 25, 25, 25}_{5}$

$$N_{tot} = \sum_{j=0}^{\infty} N_j = 14$$

$N_{25} = 5$ etc

Prob. of a person having age j is

$$P_j = N_j / N_{tot}$$

$$P_{25} = 5/14$$

By construction $\sum_{j=0}^{\infty} P_j = 1$

Note most of P_j are zero.
$P_{256} = 0$ for example.

$\boxed{\text{Most probable value}}$ j with largest $P_j \Rightarrow 25$.

$\boxed{\text{Median value}}$ j_{median}

$$\sum_{j=0}^{j_{median}} P_j = \frac{1}{2} \Rightarrow j_{median} = 23$$

7 younger
7 older

Average or Mean Value

also called expectation value

$$\langle j \rangle = \sum_j j \, P_j$$

$$= \frac{14 + 15 + 3(16) + 2(22) + 2(24) + 5(25)}{14}$$

$$= 21$$

Mean square value

$$\langle j^2 \rangle = \sum_j j^2 \, P_j = \frac{14^2 + 15^2 + 3(16)^2 + 2(22)^2 + 2(24)^2 + 5(25)^2}{14}$$

Note $\langle j^2 \rangle \neq \langle j \rangle^2$

Variance

$$\sigma^2 \equiv \langle (j - \langle j \rangle)^2 \rangle = \langle (\Delta j)^2 \rangle$$

Note $\langle \Delta j \rangle = 0$ by construction of $\langle j \rangle$.

$$\sigma^2 = \langle j^2 - 2j\langle j \rangle + \langle j \rangle^2 \rangle$$

$$= \langle j^2 \rangle - 2\langle j \rangle \langle j \rangle + \langle j \rangle^2 = \langle j^2 \rangle - \langle j \rangle^2$$

Standard deviation $\equiv \sigma$ greek sigma

$$\sigma = \left[\langle j^2\rangle - \langle j\rangle^2\right]^{1/2}$$

Contin Variable

$$P_j \longrightarrow \rho(x) \qquad \text{greek rho}$$

Normalization

$$\int_{-\infty}^{\infty} dx \; \rho(x) = 1 \qquad (\rho = \Psi^* \Psi)$$

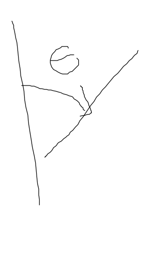
$$\langle x\rangle = \int_{-\infty}^{\infty} x \, \rho(x) \, dx$$

$$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2 \, \rho(x) \, dx$$

$$\sigma^2 = \langle(\Delta x)^2\rangle = \langle x^2\rangle - \langle x\rangle^2$$

σ = uncert. in position for H. Uncert. principle.

Example Needle thrown at random



angle θ is uniformly distributed $0 \rightarrow \pi$

$$\rho(\theta) = \text{const}$$

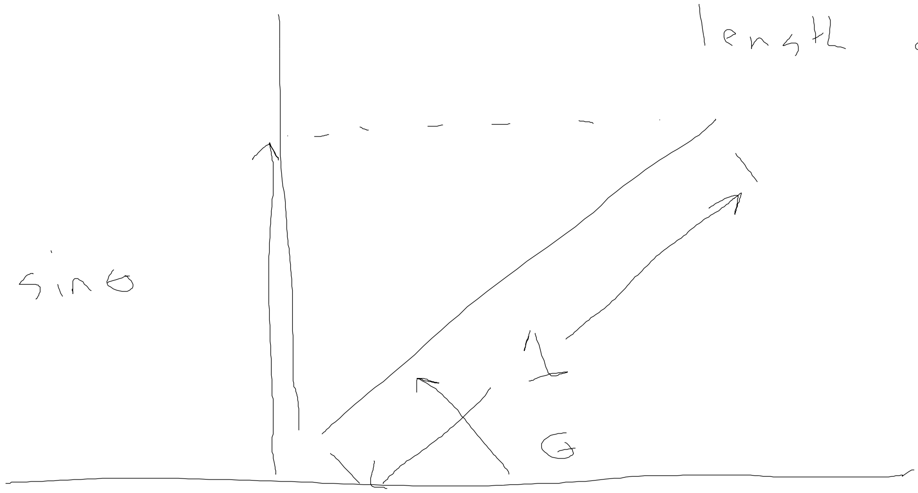
$$\int_0^\pi \rho(\theta)\, d\theta = 1$$

$$\Rightarrow \text{const.} = \pi^{-1}$$

$$\rho(\theta) = \frac{1}{\pi}$$

What is average height of needle

length of needle is one.



$$\langle \sin\theta \rangle = \int_0^\pi \sin\theta\, \rho(\theta)\, d\theta$$

$$= \frac{1}{\pi} \int_0^\pi \sin\theta\, d\theta$$

$$\langle \sin\theta \rangle = \frac{2}{\pi}$$

If I throw needle at random what is prob. it will lie on a line. Set of horizontal lines set 1 unit apart.

$$P = \langle \sin\theta \rangle = \frac{2}{\pi}$$

If I place an object of height $\langle \sin\theta \rangle$ at random it has prob. of P of crossing a vertical line.

Calculation of π

$$\pi = \frac{2}{P}$$

Throw needle ten times and try it.