Lecture #3 Probability

Last time: Time evolution from

\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi \]

I just tell you this gives \( \Psi(x,t) \) for all \( t \geq 0 \) given \( V \) and \( \Psi(x,0) \).

Prob. interp. of wave function

\[ P(x \rightarrow x+dx) = \Psi^*(x,t) \Psi(x,t) \, dx \]

Normalization

\[ \int_{-\infty}^{\infty} dx \, \Psi^*(x,t) \Psi(x,t) = 1 \]

Prob. to find particle somewhere is 1

Chose \( \Psi(x,0) \) so that

\[ \int_{-\infty}^{\infty} dx \, \Psi^*(x,0) \Psi(x,0) = 1 \]

Show later \( \frac{d}{dt} \int_{-\infty}^{\infty} dx \, \Psi^*(x,t) \Psi(x,t) = 0 \)
General background on Probability

Monte Carlo methods very important in Physics calculations. Example
Monte Carlo evaluation of an integral

\[ I = \int_{0}^{1} f(x) \, dx \approx \frac{1}{N} \sum_{i=1}^{N} f(x_i) \]

\( x_i \) are random numbers uniformly distributed on \((0, 1)\).

Some references: Press et al. "Numerical Recipes" discusses random number

Consider first a Discrete Variable
only takes on integer values

Example: room full of 14 people
of age
14, 15, 16, 16, 16  \underline{22, 22}  24, 24  \underline{25, 25, 25, 25} \\
\[ \underline{3, 2, 2} \]
\[ N_{tot} = \sum_{j=0}^{\infty} N_j = 14 \]
\[ N_{25} = 5 \quad \text{etc} \]

Probability of a person having age \( j \) is
\[ P_j = \frac{N_j}{N_{tot}} \]
\[ P_{25} = \frac{5}{14} \]

By construction

Note most of \( P_j \)
\[ P_{25} = 0 \quad \text{for} \quad j > 25 \]

Most probable value \( \Rightarrow \quad P_j \quad \Rightarrow \quad 25 \).

Median value \( J_{\text{median}} \)
\[ \sum_{j=0}^{\infty} \frac{P_j}{2} = \frac{1}{2} \quad \Rightarrow \quad J_{\text{median}} = 23 \]

7 younger
7 older
Average or Mean Value

Also called expectation value

\[ <j> = \sum j \theta_j \]
\[ = \frac{14 + 15 + 3 \cdot 14 + 2(22) + 2(24) + 5(25)}{14} \]
\[ = 21 \]

Mean Square Value

\[ <j^2> = \sum j^2 \theta_j = \frac{14^2 + 15^2 + 3 \cdot 14^2 + 2(22)^2 + 2(24)^2 + 5(25)^2}{14} \]

Note \( <j^2> \neq <j>^2 \)

Variance

\[ \sigma^2 = <(j - <j>)^2> = <(\Delta j)^2> \]

Note \( <\Delta j> = 0 \) by construction of \( <j> \).

\[ \sigma^2 = <j^2> - 2<j><j> + <j>^2 \]
\[ = <j^2> - 2j<j> + <j>^2 \]
\[ = <j^2> - 2<j><j> = <\Delta j> <j> \]
\[ <j^3> = <j^2> - <j>^2 \]
Standard deviation $\equiv \sigma$ (greek sigma)

\[ \sigma = \left[ \langle j^2 \rangle - \langle j \rangle^2 \right]^{1/2} \]

Contin. Variable

\[ \rho \rightarrow \rho(x) \] (greek rho)

Normalization

\[ \int_{-\infty}^{\infty} \rho(x) \, dx = 1 \] (\( \rho = \Psi^* \Psi \))

\[ \langle x \rangle = \int_{-\infty}^{\infty} x \rho(x) \, dx \]

\[ \langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \rho(x) \, dx \]

\[ \sigma^2 = \langle (\Delta x)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2 \]

\[ \sigma = \text{uncert in position for \textbf{Uncert. Principle}} \]

Example

Needle thrown at random angle $\theta$ is uniformly distributed $\theta \rightarrow \pi$
\[ P(\theta) = \text{const} \]
\[ \int_0^\pi P(\theta) \, d\theta = 1 \]
\[ \Rightarrow \text{const.} = \pi^{-1} \]

\[ P(\theta) = \frac{1}{\pi} \]

What is average height of needle length of needle is one.

\[ \langle \sin \theta \rangle = \int_0^\pi \sin \theta \cdot P(\theta) \, d\theta \]
\[ = \frac{\pi}{\pi} \int_0^\pi \sin \theta \, d\theta \]
\[ = \frac{2}{\pi} \]

\[ \langle \sin \theta \rangle = \frac{2}{\pi} \]
If I throw needle at random what is prob it will lie on a line set of horizontal lines set 1 unit apart.

\[ P = \sin \theta = \frac{2}{\pi} \]

If I place an object of height \( \sin \theta \) at random it has prob. of \( \frac{1}{P} \) of crossing a vertical line.

Calculation of \( \pi \)

\[ \pi = \frac{2}{P} \]

Throw needle ten times and try it.