10/23/98

Lecture #23 Final Review

Delta function pot.

$$V = -\alpha\,\delta(x)$$

B.C. at position of delta function
(1) Wavefunction is always cont.
(2) Discont. in $d\psi/dx$ (only impact of delta function)

Integrate Shrodinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi = E\psi$$

over small region from position of delta function $-\epsilon$ to $+\epsilon$. Take limit $\epsilon \to 0$

$$-\frac{\hbar^2}{2m}\left[\left.\frac{d\psi}{dx}\right|_{+\epsilon} - \left.\frac{d\psi}{dx}\right|_{-\epsilon}\right] + \int_{-\epsilon}^{+\epsilon} dx\, V(x)\,\psi(x) = \cancel{E\int_{-\epsilon}^{\epsilon} dx\,\psi(x)}$$

$E\,2\epsilon\,\psi$

$$\boxed{-\frac{\hbar^2}{2m}\left[\left.\frac{d\psi}{dx}\right|_{+\epsilon} - \left.\frac{d\psi}{dx}\right|_{-\epsilon}\right] - \alpha\,\psi(0) = 0}$$

Relation between Wave mechanics and matrix mechanics.

In <u>Wave mechanics</u> solve S. eq. in r space

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} + V(x)\Psi(x) = E\Psi(x)$$

Can always expand general wave function in an orthonormal basis

$$|\Psi\rangle = \sum_i c_i |e_i\rangle$$ abstract state vector

Project on coordinate space basis

$$\Psi(x) = \langle x|\Psi\rangle \qquad e_i(x) = \langle x|e_i\rangle$$

$$\langle e_i|e_j\rangle = \delta_{ij} = \int dx\, e_i^*(x)\, e_j(x)$$

start with

$$\hat{H}|\Psi\rangle = E|\Psi\rangle$$

project out $\langle e_j|$ matrix element (Take dot product with $|e_j\rangle$

$$\langle e_j|\hat{H}\Psi\rangle = E\langle e_j|\Psi\rangle$$

Use expansion for $|\Psi\rangle$

$$\langle e_j | \hat{H} \sum_i c_i | e_i \rangle = E \langle e_j | \sum_i c_i e_i \rangle$$

Hamiltonian matrix

$$H_{ji} \equiv \langle e_j | \hat{H} e_i \rangle = \int dx\, e_j^*(x)\, \hat{H}\, e_i(x)$$

State vector $\quad C = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ \vdots \end{pmatrix}$

Schrodinger eq. is equiv. to matrix equation

$$[H]\,[C] = E\,[C]$$

Normalization

$$\langle \Psi | \Psi \rangle = 1$$

Wave mech.

$$\int dx\, \Psi^*(x)\, \Psi(x) = 1$$

Matrix mech.

$$C^\dagger C = \sum_i c_i^* c_i = 1$$

What did we use

(a) Completness: can expand any function in a complete set.

(b) Orthogonality $\langle e_i | e_j \rangle = 0 \quad i \neq j$

(c) Normalization. The basis states are normalized

$$\langle e_i | e_i \rangle = 1$$

so the expansion coef. are also normalized

$$\langle \Psi | \Psi \rangle = 1 \Rightarrow$$

$$c^{\dagger} c = 1$$