10/21

Lecture #22 Review

Postulates of QM

1) <u>State</u> of a particle represented by a normalized vector $|\Psi\rangle$ in the Hilbert space L_2.

Normalization $\langle\Psi|\Psi\rangle = 1$

$= \int \Psi^*(x)\Psi(x)\,dx$ in coordinate space

$= \int \Psi^*(p)\Psi(p)\,dp$ in momentum space

$= \sum_i a_i^* a_i$ as a vector, if

$|\Psi\rangle = \sum_i a_i |e_i\rangle$

[L_2 all functions such $\int dx\,|f(x)|^2 < \infty$]

2) <u>Observable</u> quantities Q are represented by Hermitian operators $\hat{Q}$. The expectation value of Q in state $|\Psi\rangle$ is

$\langle Q\rangle = \langle\Psi|\hat{Q}\Psi\rangle$

[Hermitian $\langle\Psi|\hat{Q}\Psi\rangle = \langle\hat{Q}\Psi|\Psi\rangle$]

Know what expectation value means

3) A measurement of Q on state $|\Psi\rangle$ is ~~cetain~~ to return λ iff $|\Psi\rangle$ is an eigenvector of $\hat{Q}$

$$\hat{Q}\,|\Psi_\lambda\rangle = \lambda\,|\Psi_\lambda\rangle$$

3') A measurement of Q is cetain to get one of the eigenvalues λ of Q. Prob. $P_\lambda = |\langle\Psi_\lambda|\Psi\rangle|^2$

Eigenvectors of any operator make up a complete set. Can always expand any state vector

$$|\Psi\rangle = \sum_\lambda c_\lambda\,|\Psi_\lambda\rangle$$

Can chose the set $|\Psi_\lambda\rangle$ to be orthonormal

$$\langle\Psi_{\lambda'}|\Psi_\lambda\rangle = \delta_{\lambda'\lambda}$$

Project out coef.

$$\langle\Psi_{\lambda'}|\Psi\rangle = c_{\lambda'}$$

10/21

Uncetainty Principle

Variance of a quantity

$$\sigma^2 \equiv \langle (\Delta Q)^2 \rangle = \langle \hat{Q}^2 \rangle - \langle \hat{Q} \rangle^2$$

Standard deviation (or uncertainty) is σ the square root of the variance.

If two operators $\hat{A}$ and $\hat{B}$ do not commute $[\hat{A}, \hat{B}] \neq 0$ then they can not both be known with arbitrary prec.

$$\sigma_A^2 \, \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2$$

Example $\hat{x} = x$ $\hat{p} = \frac{\hbar}{i} \frac{\partial}{\partial x}$

$[\hat{x}, \hat{p}] = i\hbar$ so

$$\sigma_x^2 \, \sigma_p^2 \geq (\hbar/2)^2$$

Time dependence in QM

State vector evolves with time according to Schrodinger eq.

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \hat{H} |\Psi\rangle$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}$$

<u>Stationary</u> states are eigenstates of $\hat{H}$

$$\hat{H} |\Psi_E\rangle = E |\Psi_E\rangle$$

Time dep. is just a phase

$$\Psi_E(x,t) = \Psi_E(x,0)\, e^{-iEt/\hbar}$$

General time dep.

$$\Psi(x,t) = \sum_i c_i \psi_i(x)\, e^{-iE_i t/\hbar}$$

Expansion coef. c_i indep. of time

<u>Solution of time dep. sch. eq.</u>

$$\hat{H} \psi(x) = E \psi(x)$$

Adjust E to satisfy boundary conditions

(i) Wave func. is always cont.

(ii) $\partial\psi/dx$ is cont. except where $V \to \infty$

(iii) $\psi(x) \to 0$ as $x \to \pm\infty$ so that
$\int dx\, \psi^* \psi < \infty$

General power series solution (example $V = \frac{1}{2}kx^2$)

(1) Pull off asym. form. Keep largest terms as $x \to \pm\infty$ and solve simple eq.

(2) $$\psi(x) = h(\xi)\, e^{-\xi^2/2}$$

asym. form

For Harmonic osc. $\xi = \sqrt{\frac{m\omega}{\hbar}}\, x$

Expand remainder $h\xi$ in a power series

$$h = \sum_i a_i \xi^i$$

(3) Adjust E to satisfy B.C. Power series must terminate to keep ψ normalizabl

$$\Longrightarrow \quad E = \left(n + \tfrac{1}{2}\right)\hbar\omega$$

(4) Normalize resulting solution $\int_{-\infty}^{\infty} dx\, \psi_n^* \psi_n = 1$

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}}\, H_n(\xi)\, e^{-\xi^2/2}$$

H_n = Hermite polynomials

10/21

Review other solutions

(a) Particle in a box

(b) Finite square well

(c) Delta function pot.

(d) Free particle and Fourier transform