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Lecture #35 Bosons and blackbody radiation

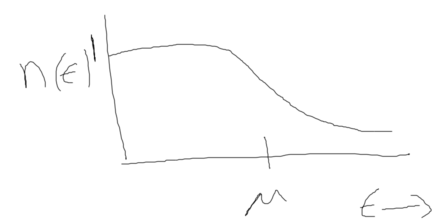
Chose fractional occupation $N_i/d_i = n(\epsilon)$ in such a way as to maximiz # of possible state subject to constraints

$$N_{Tot} = \sum_i N_i$$
$$E_{tot} = \sum_i{}' E_i N_i$$

For fermions # of ways to put N_i fermions into d_i states $= \binom{d_n}{N_n}$

Result is Fermi-Dirac dist.

$$n(\epsilon) = \left[e^{(\epsilon-\mu)/kT} + 1\right]^{-1}$$



For Bosons can put more then one boson into a given state. # of ways to put N_i bosons into d_i levels $= \binom{N_i+d_i-1}{N_i}$

See text. This gives Bose-Einstien distribution

$$n(\epsilon) = \left[e^{(\epsilon-\mu)/kT} - 1\right]^{-1}$$

Note minus sign. For small $\epsilon-\mu$ $n(\epsilon)$ can be large then one.

Chose μ to set corect N_{tot}

$$N_{tot} = \sum_i d_i \; n(E_i)$$

From last lecture $d_i = \frac{V}{2\pi^2} k^2 dk$

$$\boxed{N_{tot} = \frac{V}{2\pi^2} \int_0^\infty k^2 dk \; n(E_k)}$$

Density $\rho = N_{tot}/V$, For free particles

$$\rho = \frac{1}{2\pi^2} \int_0^\infty k^2 dk \left[\frac{1}{e^{\left(\frac{\hbar^2 k^2}{2m} - \mu\right)/kT} \pm 1} \right]$$

Integral does not exist in closed form but can invert numerically to find $\mu(\rho, T)$

For Fermions μ can be negative (low density) or positive (high density)

For Bosons μ must be negative or zero. It can never be positive.
Consider $n(\epsilon = 0)$

$$n(0) = \left[e^{-\mu/kT} - 1 \right]^{-1}$$

This would be negative for positive μ. Thus $\mu \leq 0$. For bosons this implies an upper limit on the density

$$\rho \leq \rho_{max} = \frac{1}{2\pi^2}\int_0^\infty k^2 dk \left[\frac{1}{e^{\frac{\hbar^2 k^2}{2m}/k_B T} - 1}\right]$$

set $\mu = 0$

If one increases the density above ρ_{max} the extra particles all condense into the $E=0$ state. This state has a macroscopic occupation #. This is called Bose-Einstien condensation and has recently been observed in gas systems cooled to very low temperatures. It is also related to the superfluid state in liquid He below $\sim 2K$. [See problem 5.26]

Note if one state is very important then our approx of replacing sum by integral fails for $N=0$ state.

Black body radiation

Free Photons gas

(a) $E = h\nu = \hbar\omega$

(b) $k =$ wave # $= 2\pi/\lambda = \omega/c$

(c) Only two spin states. Photons are spin $l=1$ so expect $m=-1,0,1$ but $m=0$ state ruled out by relativity for massless photons

(d) The number of photons is not conserved. Thus $\mu = 0$.

Density of photons with $E = \hbar\omega$

$$n(\omega) = \left[e^{\frac{\hbar\omega}{k_B T}} - 1\right]^{-1}$$

Energy density of photons with frequency within $d\omega$ of ω is

$$E(\omega) = \frac{N(\omega)}{V}\hbar\omega \, d\omega$$

$$N(\omega) = d_\omega \, n(\omega)$$

spin states

$$d_\omega = 2V/_{\pi^2} \, k^2 dk$$

$$k = \omega / c$$

$$d_\omega = \frac{V}{\pi^2} \frac{\omega^2 d\omega}{c^3}$$

$$E(\omega) = \frac{1}{V} \frac{\omega^2 d\omega}{c^3} \frac{V}{\pi^2} \hbar\omega \left[e^{\frac{\hbar\omega}{kT}} - 1 \right]^{-1}$$

Define $E(\omega) = \rho(\omega)\, d\omega$

$$\boxed{\rho(\omega) = \frac{\hbar\, \omega^3}{\pi^2 c^3 \left[e^{\hbar\omega / k_B T} - 1 \right]}}$$

Planck's <u>blackbody spectrum</u> giving energy per unit volume per unit frequency in an electromagnetic field at equilibrium temperature T.

$\rho(\omega)$