Lecture #39  Review

Quantum mechanics describes the motion of small objects. A measurement must disturb a system to some level.

\[ \Delta p \Delta x \geq \frac{\hbar}{2} \]

Since \( p = mV \) we can determine both the velocity and position of massive objects

\[ \Delta V \Delta x \geq \frac{\hbar}{2m} \rightarrow 0 \]

Thus QM reduces to classical mech. for massive objects. Correspondence principle in limit of large quantum number. QM system corresponds with any classical. Example 1 dim HO

Ground state (pure QM)

Largen prob. dist-close to classical
Postulates of QM

1) The state of a system is represented by a normalized vector, $|\Psi\rangle$ (wave function).

2) Observable quantities $Q(x, p, t)$ are represented by Hermitian operators $\hat{Q}$. The expectation value of $Q$ is $\langle Q \rangle = \langle \Psi | \hat{Q} | \Psi \rangle$.

3) A measurement of the observable $Q$ on a system in the state $|\Psi\rangle$ is certain to yield $\lambda$ if $|\Psi\rangle$ is an eigenvector of $\hat{Q}$ with eigenvalue $\lambda$, $\hat{Q} |\Psi\rangle = \lambda |\Psi\rangle$.

3') A measurement of $Q$ in the state $|\Psi\rangle$ is certain to get one of the eigenvalues of $\hat{Q}$. The prob. of getting $\lambda$ is equal to the absolute square of the $\lambda$ component of $|\Psi\rangle$.

Eigenvectors $\hat{Q} |\Psi\rangle = \lambda |\Psi\rangle$

Completeness can expand any $|\Psi\rangle$

$|\Psi\rangle = \sum \langle x | |\Psi\rangle |x\rangle$

Expansion coef. $\langle x | = \langle x | |\Psi\rangle |x\rangle$
Orthogonality

\[ \langle \chi_i | \chi_j \rangle = \delta_{ij} \]

Prob. of getting \( \lambda \)

\[ P_\lambda = |c_\lambda|^2 \]

Expectation value

\[ \langle \hat{\mathcal{Q}} \rangle = \langle \hat{\mathcal{Q}} \rvert \Psi \rangle \]

\[ = \sum \mathcal{P}_\lambda \lambda \]

Generalized Uncert Principle

\[ \sigma_A = \sqrt{\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2} \quad \text{Variance} \]

\[ \sigma_A = \text{Standard deviation} \]

\[ \sigma_A \sigma_B \geq \frac{1}{2} \sqrt{\langle [\hat{A}, \hat{B}] \rangle} \]

\[ [\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \quad \text{Commutator} \]