3/19/01

Lec. 25 Angular Momentum

H Spectrum

$$E_\gamma = h\nu = E_i - E_f = -\frac{m_e c^2}{2}\alpha^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$E_1 = -13.6\,eV = -\frac{m_e c^2}{2}\alpha^2$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

$$\nu = c/\lambda$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

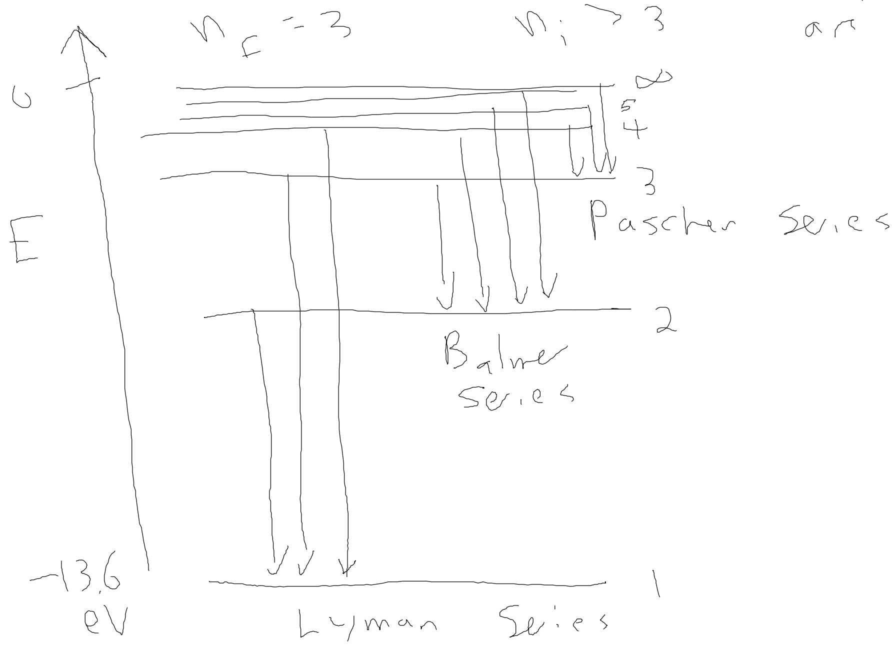
$$R = \frac{m_e}{4\pi c \hbar^3}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 = 1.097\times10^7\ m^{-1}$$

Rydberg constant

$n_f = 1$ $\quad n_i > 1$ are in Ultraviolet (Lyman Series)

$n_f = 2$ $\quad n_i > 2$ are in Visable (Balmer Series)

$n_f = 3$ $\quad n_i > 3$ are in infrared (Paschen Series)



Deuterium Spectrum

Big bang produced mostly ^1H about 25% ^4He and a small amount of ^2H (deuterium), ^3He and Li

Ratio of D/H from big bang provides crucial information on density of matter in Universe.

Higher the density more D gets burned into heavier ^3He + Li. and lower D abundance.

How does deuterium spectrum differ from ^1H spectrum? $\Rightarrow$ Reduced mass effects from recoil of nucleus

Have two body problem for $\vec{r}_n = \vec{r}_{nucleus}$ and $\vec{r}_e$

In classical mechanics particles orbit common center of mass. Behaves like single particle with

$$\mu = \text{reduced mass} = \frac{m_e m_p}{m_e + m_p} \simeq m_e \quad \text{for } {}^1H$$

$$\mu_H \simeq m_e \left(1 - \frac{m_e}{m_p}\right)$$

$$\simeq m_e \left(1 - \frac{1}{2000}\right)$$

$$\mu_D \simeq m_e \left(1 - \frac{m_e}{m_d}\right) \simeq m_e \left(1 - \frac{1}{4000}\right)$$

Since $m_d \simeq m_n + m_p \simeq 2 m_p$

See problem 5.1 p 178

Since $E_i \propto \mu$

The energy of deterium states are about one part in 4000 larger than the energy of ^1H states.

$$E_i(d) = -\frac{m_e c^2}{2}\alpha^2 \left(1 - \frac{m_e}{m_d}\right)\frac{1}{n_i^2}$$

$$= -\frac{\mu c^2}{2}\alpha^2 \frac{1}{n_i^2}$$

$$E_i(^1H) = -\frac{m_e c^2 \alpha^2}{2}\left(1 - \frac{m_e}{m_p}\right)\frac{1}{n_i^2}$$

Therefor decterium spectral lines have shorte wavelengths by one part in 4000.

<u>Example</u> First Balme series line has

$$\lambda^{-1} = 1.097 \times 10^7 \text{ m}^{-1}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\lambda = 6563 \text{ Å}$$

(i.e. Red)

First Lyman Series line

$$\lambda^{-1} = 1.097 \times 10^7 \text{ m}^{-1}\left(1 - \frac{1}{4}\right)$$

$\lambda = 1215$ Å Ultraviolet

In a cloud at $z = 0.701$ this light is red shifted so final wave length is

$$\lambda = (1+z)\ 1215\ \text{Å}$$
$$= 2066.7\ \text{Å}$$

Correct for center of mass

$$\lambda(H) = 2066.7 \Big/ \left(1 - \frac{1}{2000}\right) = 2067.7\ \text{Å}$$
$$\lambda(d) = 2066.7 \Big/ \left(1 - \frac{1}{4000}\right) = 2067.2\ \text{Å}$$
$$\lambda(d) - \lambda(H) = 0.5\ \text{Å}$$

Results

$$d/H \sim 2 \times 10^{-5}$$

Angular momentum

$$\vec{L} = \vec{r} \wedge \vec{p}$$
$$L_x = y p_z - z p_y = \frac{\hbar}{i}\left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y}\right)$$
$$L_y = z p_x - x p_z = \frac{\hbar}{i}\left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z}\right)$$
$$L_z = x p_y - y p_x = \frac{\hbar}{i}\left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}\right)$$

Commutator of L_x with L_y

$$[L_x, L_y] = \left(\frac{\hbar}{i}\right)^2 \left[\left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y}\right)\left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z}\right)\right.$$

$$-\left(z\frac{\partial}{\partial x}-x\frac{\partial}{\partial z}\right)\left(y\frac{\partial}{\partial z}-z\frac{\partial}{\partial y}\right)$$

$$=\left(\frac{\hbar}{i}\right)^2\left[y\frac{\partial}{\partial x}-x\frac{\partial}{\partial y}\right]$$

$$=\frac{\hbar}{i}\left(-\frac{\hbar}{i}\right)\left[x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right]$$

$$=i\hbar L_z$$

$$\boxed{\begin{aligned}[L_x,L_y]&=i\hbar L_z\\ [L_y,L_z]&=i\hbar L_x\\ [L_z,L_x]&=i\hbar L_y\end{aligned}}$$

Can't simultaniously know L_x and L_y

$$\sigma^2_{L_x}\,\sigma^2_{L_y}\geq\left(\frac{1}{2i}\langle i\hbar L_z\rangle\right)^2=\frac{\hbar^2}{4}\langle L_z\rangle^2$$

Instead consider L^2 and L_x

$$[L^2,L_x]=[L_y^2,L_x]+[L_z^2,L_x]$$

with $L^2=L_y^2+L_x^2+L_z^2$

note $[L_x^2,L_x]=0$

$$[L_y^2,L_x]=L_y[L_y,L_x]+[L_y,L_x]L_y$$

$$=-i\hbar L_yL_z-i\hbar L_zL_y$$

$$[L_z^2, L_x] = L_z[L_z, L_x] + [L_z, L_x]L_z$$

$$= i\hbar L_z L_y + i\hbar L_y L_z$$

So $[L^2, L_x] = -i\hbar L_y L_z - i\hbar L_z L_y$

$$+ i\hbar L_z L_y + i\hbar L_y L_z = 0$$

$$\boxed{\begin{aligned} [L^2, L_x] &= 0 \\ [L^2, L_y] &= 0 \\ [L^2, L_z] &= 0 \end{aligned}}$$