Lec. 24 Hydrogen Spectrum

General wave function

\[ \psi_{nlm} = \frac{1}{\sqrt{P_n}} e^{-\rho} \psi(r) R_{nlm}(\theta, \phi) \]

Radial wave function depends on both n and l. Energy only depends on n. This is an accident of the potential. In general, we expect E to depend on both n and l (but not m).

Degeneracy

For a given n and l, one can go from 0 to n-1. For each allowed m, values range from -l to l. For a total of 2l+1 values.

Degeneracy of nth energy level

\[ g(n) = \sum_{l=0}^{n-1} (2l+1) = n^2 \]

Example: n=1

\[ R_{10} = 2a^{-3/2} e^{-r/a} \]
\[ R_{20} = \frac{1}{\sqrt{52}} a^{-3/2} \left( 1 - \frac{1}{2} \frac{r}{a} \right) e^{-r/2a} \]
\[ R_{21} = \frac{1}{\sqrt{52}} a^{-3/2} \left( \frac{r}{a} \right) e^{-r/2a} \]

R_{20} needs one node to be orthogonal to R_{10} however R_{21} does not need to be.
to be orthogonal to $R_{10}$ because
the $Y_m$'s are orthogonal

$\int_{R_{10}}^\infty R_{21} R_{10} \, d\alpha \neq 0$

but $\int_{R_{10}}^\infty \sum_{m=0}^{\infty} Y_m \cdot Y_{10} \, d\alpha = 0$

**Spectrum of Hydrogen**

If an atom is in an excited state
by emission of a photon

$E_Y = h\nu = E_i - E_f = -13.6\text{eV} \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$

$E_i = -13.6\text{eV} = -\frac{m_e c^2}{2} \alpha^2 = -0.511 \times 10^6\text{eV} \left( \frac{1}{137} \right)$

$\nu = \frac{c}{\lambda}$

Wave length

$R = \frac{m_e c^2}{\nu^2} \left( \frac{e^2}{4\pi \varepsilon_0} \right)^2 = \frac{1,097 \times 10^7}{n_i^2} \text{m}^{-1}$

Rydberg Constant

Transitions to $n_f = 1$ from $n_i > 1$ are in
Ultra violet (Lyman series)

Transitions to $n_f = 2$ from $n_i > 2$ are in
Visible (Balmer series)

Transitions to $n_f = 3$ from $n_i > 3$ are in
Infrared (Paschen series)