3/7/01

Lec. 24 Hydrogen Spectrum

General wave function

$\nu(\rho)$ is a polynomial of degree $n-l-1$

$$\psi_{nlm} = \underbrace{\frac{1}{r}\rho^{l+1} e^{-\rho} \nu(\rho)}_{R_{nl}(r)} Y_l^m(\theta,\phi)$$

Radial wave function depends on both n and l. Energy only depends on n. This is an accident of $1/r$ pot. In general expect E to depend on both n and l (but not m)

Degeneracy

For a given n l can go from 0 to $n-1$. For a given l the allowed m values range from $-l$ to l for a total of $2l+1$ values

Degeneracy of n^{th} energy level

$$d(n) = \sum_{l=0}^{n-1} (2l+1) = n^2$$

Example $n=1$

$$R_{10} = 2a^{-3/2} e^{-r/a}$$

$$R_{20} = \frac{1}{\sqrt{2}} a^{-3/2}\left(1 - \frac{1}{2}\frac{r}{a}\right) e^{-r/2a}$$

$$R_{21} = \frac{1}{\sqrt{24}} a^{-3/2}\left(\frac{r}{a}\right) e^{-r/2a}$$

R_{20} needs one node to be orthogonal to R_{10}. However R_{21} does not need to be orthogonal to R_{10} because the Y_l^m's are orthogonal

$$\int_0^\infty R_{10}(r)\, R_{21}(r)\, r^2 dr \neq 0$$

but

$$\int_0^{2\pi} d\phi \int_0^\pi \sin\theta\, d\theta\, Y_1^{m*}\, Y_0^0 \equiv 0$$

Spectrum of Hydrogen

If an atom in an excited state E_i decays to the state E_f by emission of a photon

$$E_\gamma = h\nu = E_i - E_f = -13.6\,\text{eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$E_1 = -13.6\,\text{eV} = -\frac{m_e c^2}{2}\alpha^2 = -\frac{0.511\times10^6\,\text{eV}}{2}\left(\frac{1}{137}\right)^2$$

$\nu = c/\lambda$ wave length

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

Rydberg formula

$$R = \frac{m}{4\pi c \hbar^3}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 = 1.097\times10^7\ \text{m}^{-1}$$

Rydberg Constant

Transitions to $n_f = 1$ from $n_i > 1$ are in Ultra violet (Lyman Series)

Transitions to $n_f = 2$ from $n_i > 2$ are in visable (Balme Series)

Transitions to $n_f = 3$ from $n_i > 3$ are in infrared (Paschen Series)