Lecture #3 Probability 1/14/00

Last time watched a wave packet move and spread

\[ \Psi = e^{-\frac{(x-X_0(t))^2}{2(\Delta x(t))^2}} e^{\frac{i x}{\lambda}} \]

e \equiv \cos \alpha + i \sin \alpha

Complex oscillated phase makes particle move

\[ k = \frac{\Delta x}{\lambda} \]

\[ \Delta x(t) = \text{Width of gaussian} \]

\[ \Delta x(t) = \left[ (\Delta x(t=0))^2 + \left(\frac{\Delta p}{m}\right)^2 \right]^{1/2} \]

Width spreads because of uncertainty in momentum

\[ X_0(t) = X_0 + \frac{p}{m} t \]

Probability in Quantum Mechanics

\[ |\Psi(x,t)|^2 dx = \text{Prob. to find particle between } x \text{ and } x+dx \text{ at time } t \]
Suppose I do make a measurement and find the particle at C.

Question: Where was the particle just before I made the measurement?

1) Realist: Einstein

The particle was at C, QM wave function which only has a prob. For being at C and nonzero prob. For being else where is incomplete.

H.U.P. is statement about our ignorance of the particle's position rather than a statement that the particle does not have a well defined position until it is measured.

2) Orthodox: Bohr

The particle wasn't really anywhere. It was the act of measurement that forced the particle to "take a stand."

"Observations not only disturb what is to be measured they produce it."

View called Copenhagen interpretation

From Niels Bohr inst. in Copenhagen no physical theory can tell where the particle is anymore then the QM description

Note: measurement is an important part of QM.

Consider politics after a roll call vote.
causes a politician to have a position.

Probability plays central role in QM.

Example: Monte Carlo integration

Can think of integral as related to average value of a function

\[ I = \int_0^N x \cdot f(x) = \frac{1}{N} \sum_{i=1}^{N} f(x_i) \]

\( x_i = \) random number \( \text{(uniformly distributed on } 0 \text{ to } 1) \)

In Basic (QBASIC, EXE) Rnd returns a random #

Example

\[ I = \int_0^1 x^n = \frac{x^{n+1}}{n+1} \bigg|_0^1 = \frac{1}{n+1} \]

\[ \text{Integral} = c \]
\[ \text{For, } I = 1 \text{ to } N+1 \]
\[ \text{Integral} = \text{Integral} + \text{Rnd}^n \]
\[ \text{Next I} \]
\[ \text{Integral} = \text{Integral} / N+1 \]

The larger \( N+1 \), the more accurate.

The integration
If the integral is from $a$ to $b$ we can just change variables

$$z_i = a + (a-b) x_i$$

$x_i$ uniform on $[a,b]$

$z_i$ uniform on $[a,b]$.

$$I_{ab} = \int_a^b dz \ f(z) = \frac{1}{N_{tot}} \sum_{i=1}^{N_{tot}} f(z_i)$$

Note $a-b = (a-b) \langle f \rangle$

Example: Gaussian integral will be important in what follows.

$$I = \int_{-\infty}^{\infty} dx \ e^{-x^2}$$

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Let $a = -10 \ , \ b = 10$ \ [ $\int_{-10}^{10} e^{-x^2} \ dx$ is small]

$$I = \frac{20}{\sqrt{\pi}} \sum_{i=1}^{N_{tot}} e^{-z_i^2}$$

Trick: Write not $I$ but $I^2$ and change dummy variables in $I^2$ and integral.

$$I^2 = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \ e^{-(x^2 + y^2)}$$

Now go to polar coordinates.

$$x = r \cos \theta \ \ \ y = r \sin \theta \ \ \ \ \ r^2 + y^2 = \rho$$

$$dx \ dy = r dr \ d\theta$$

$$x^2 + y^2 = \rho^2$$
\[ I^2 = \int_0^\infty r \, dr \int_0^{2\pi} d\theta \, e^{-r^2} \]
\[ \int_0^{2\pi} d\theta = 2\pi \]
\[ I^2 = 2\pi \int_0^\infty r \, dr \, e^{-r^2} \]
\[ t = r^2 \quad \frac{dt}{2} = r \, dr \]
\[ I^2 = 2\pi \int_0^\infty \frac{dt}{2} \, e^{-t} = \pi \]
\[ I = \pi^{1/2} \approx 1.772 \]

Example
Consider a room with people of different ages:
- \[ N(4) = 1 \]
- \[ N(5) = 1 \]
- \[ N(6) = 3 \]
- \[ N(22) = 2 \]
- \[ N(25) = 5 \]

Total # of people in room:
\[ N_{tot} = \sum_{j=0}^{8} N(j) = 14 \]

Note: most \( N(j) = 0 \)

What is the prob. of age \( j \)?
\[ P(j) = \frac{N(j)}{N_{tot}} \]
\[ \sum P(j) = 1 \]

1) Most probable age largest \( P_j \) \( \Rightarrow \) 25
Most probable position in QM is where largest value of \( \psi \) is

2) Median
50% of people are younger than \( j \) and 50% are older
\[ j = 23 \] has 7 people younger and 7 people older

3) Mean value or average value,
\[ \langle j \rangle = \sum_j j P(j) = 21 \]
\[ \langle j \rangle = \frac{14 + 15 + 3.16 + 2.22 + 2.24 + 5.25}{14} \]
In QM \[ \langle X \rangle = \int dx x \psi \star \psi \]

Note in statistics or house prices often quote median or average price rather than average price of houses so a few very expensive houses don’t skew average.
4) What is average square of age

\[ \langle j^2 \rangle = \sum_j j^2 \ p(j) \]

In general, can take expectation value of any function

\[ \langle F(j) \rangle = \sum_{j=0}^\infty F(j) \ p(j) \]

\[ \langle F(x) \rangle = \int_{-\infty}^{\infty} F(x) \ p(x) \ dx \]