1/10/00

Lecture 2 Probability

Basic quantum notion

Wave function describes system

$\Psi(x,t)$ = Probability amplitude

Wave function by itself has no physical interp. However square of wave function does

$$|\Psi(x,t)|^2 = \rho(x,t) = \text{Prob. density}$$

Probability density = Probability per unit length.

Probability to find the particle described by $\Psi(x,t)$ between x and $x + dx$ is

$$\rho(x,t)\,dx = \text{Probability} = |\Psi|^2 dx$$

Normalization = Prob. to find particle somewhere is one.

$$\boxed{\int_{-\infty}^{\infty} \rho(x,t)\,dx = 1}$$ for all time

Review Probability

Read Chap. 1 of Griffiths 1.1 - 1.4

Probability to get result j = P_j

Normalization $\sum_{j=0}^{\infty} P_j = 1$

(Note assume discrete variable j)

Prob. has two properties $P_j \geq 0$ and $\sum_j P_j = 1$

Probability that something (anything) happens is 1

Example: role one fair die

$$P_1 = P_2 = \cdots P_6 = 1/6$$

$$P_0 = P_j = 0 \qquad j > 6$$

Average or Mean value (expectation value)

$$\langle j \rangle \equiv \sum_{j=0}^{\infty} j \, P_j = (1+2+3+4+5+6)\frac{1}{6} = 3.5$$

Average of some function of j

$$\langle f(j) \rangle = \sum_{j=0}^{\infty} f(j) \, P_j$$

Example $\langle j^2 \rangle = \sum_j j^2 P_j$

$$= (1+4+9+16+25+36)\frac{1}{6}$$

$$\langle j^2 \rangle = 15.1666$$

What is best guess if you role one die for the results?

Mean value $\simeq 3.5 = \langle j \rangle$

This at least tells you that you are not likely to get -6 or $+123$

Uncertainty or spread in results

$$\Delta j = j - \langle j \rangle$$

Of course by definition of $\langle j \rangle$

$$\langle \Delta j \rangle = \langle j - \langle j \rangle \rangle = \langle j \rangle - \langle j \rangle = 0$$

Try

$$(\Delta j)^2 = j^2 - 2j\langle j \rangle + \langle j \rangle^2$$

$$\sigma^2 \equiv \langle (\Delta j)^2 \rangle = \langle j^2 \rangle - 2\langle j \langle j \rangle \rangle + \langle j \rangle^2$$

$$= \langle j^2 \rangle - 2\langle j \rangle \langle j \rangle + \langle j \rangle^2 =$$

$$\boxed{\sigma^2 = \langle (\Delta j)^2 \rangle = \langle j^2 \rangle - \langle j \rangle^2}$$

$\sigma^2 \equiv$ Variance

$$\sigma = \langle (\Delta j)^2 \rangle^{1/2} = \left[\langle j^2 \rangle - \langle j \rangle^2 \right]^{1/2}$$

$\sigma \equiv$ Standard deviation

Example for die $\quad \sigma^2 = 15.1666 - (3.5)^2$

$$\sigma^2 = 2.917$$

$$\boxed{\sigma = 1.708}$$

Standard deviation is a measure of the uncertainty in the quantity j that is used in the Heisenberg Uncertainty principle

About 69% of the time results for j are between

$$\langle j \rangle - \sigma = 3.5 - 1.71 = 1.8$$

and

$$\langle j \rangle + \sigma = 3.5 + 1.71 = 5.2$$

Our best guess for the result is

$$\langle j \rangle \pm \sigma$$

Example for die only 1 and 6 are outside of 3.5 ± 1.7 so

$\frac{4}{6}$ of the time $- 66\%$

results are within 1σ of mean

Bell Curve of test scores

In general area in high plus low tails is $31\% = 1 - .69$

$\Rightarrow$ About 30% of time result of a measurement is slightly outside $\langle j \rangle \pm \sigma$

For a continuous variable such as position replace sum by integral

Normalization $\quad 1 = \int_{-\infty}^{\infty} P(x)\, dx$

mean value $\quad \langle x \rangle = \int_{-\infty}^{\infty} x\, P(x)\, dx$

average of some function

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x)\, P(x)\, dx$$

Uncertainty in position

$$\sigma^2 = \langle (\Delta x)^2 \rangle = \underbrace{\int x^2 P(x)\,dx}_{\langle x^2 \rangle} - \underbrace{\left[\int x\, P(x)\,dx\right]^2}_{\langle x \rangle^2}$$

$$\Delta x = \sigma = \left[\langle x^2 \rangle - \langle x \rangle^2\right]^{1/2}$$

Note

$$P(x) \Rightarrow |\Psi(x,t)|^2$$

In general prob. density depends on time

For a free particle expect Δx to grow with time because of uncertainty in velocity $\Delta v \sim \frac{\Delta p}{m}$

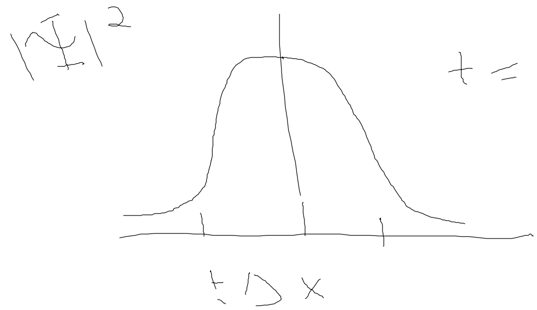
From $x(t) = x_0 + \frac{p}{m} t$

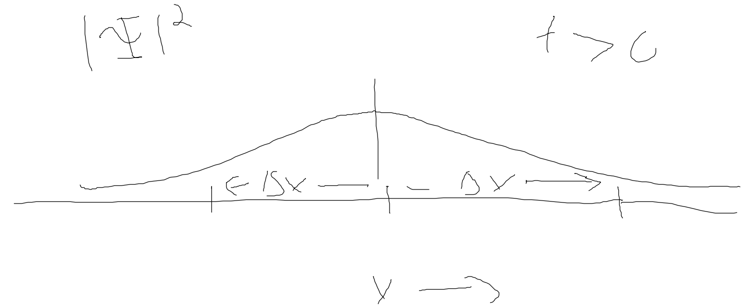
assume errors in position and momentum are uncorrelated

$$\Delta x(t) \sim \left[\Delta x(t=0)^2 + \left(\frac{\Delta p}{m} t\right)^2\right]^{1/2}$$

$$\geq \Delta x(t=0)$$

Wave function spreads with time





Note area under each curve is one.

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