12/11/98

Lecture #40 Final Review

1) Time dependence in QM given by

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi$$

2) Look for stationary states

$$\Psi(x,t) = \psi(x)\, e^{-iEt/\hbar}$$

$$\hat{H}\psi(x) = E\psi(x)$$

3) Nontrivial time dependence only for superposition of stationary states

$$\Psi(x,t) = \sum_i c_i \psi_i(x)\, e^{-iE_i t/\hbar}$$

Can always expand any wave function in stationary states

4) Solution to 1 dim $\hat{H}\psi(x) = E\psi(x)$

a) Wave function is always cont.

b) Derivative $\frac{d\psi}{dx}$ is cont except where $V \to \infty$

c) Adjust E to satisfy boundary conditions

$$\psi(x) \to 0 \quad \text{as} \quad x \to \pm\infty$$

So that $\int dx\, \psi^* \psi < \infty$

5) General analytic solution of 1 dim S. eq.
Example H. osc.

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + \frac{1}{2}m\omega^2 x^2 \psi = E\psi$$

$$\xi = \sqrt{m\omega/\hbar}\; x \qquad K = 2E/\hbar\omega$$

$$\frac{\partial^2 \psi}{\partial \xi^2} = (\xi^2 - K)\psi$$

a) Find large ξ solution $\frac{\partial^2 \psi}{\partial \xi^2} = \xi^2 \psi \quad \psi \propto e^{-\xi^2/2}$

$$\Rightarrow \psi = h(\xi)\, e^{-\xi^2/2}$$

b) Expand $h(\xi)$ in power series $h = \sum_i a_i \xi^i$

c) adjust E so wave function is normalizable (series terminates) $E = (n+\frac{1}{2})\hbar\omega$

Three dimensions

Seperation of variables for spherical sym. pot.

$$V(\vec{r}) \to V(r)$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \phi^2}$$

$$\psi(r,\theta,\phi) = R(r)\, Y(\theta,\phi)$$

Angular eq.

$$\sin\theta \frac{\partial}{\partial\theta} \sin\theta \frac{\partial}{\partial\theta} Y + \frac{\partial^2 Y}{\partial\phi^2} = -l(l+1)\sin^2\theta\, Y$$

$$Y = Y_l^m(\theta,\phi) \quad \text{spherical Harmonic}$$

$$\int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi\, Y_{l'}^{*m'}\, Y_l^m = \delta_{mm'}\,\delta_{ll'}$$

$$R(r) = u(r)/r \quad \text{reduced radial wave function}$$

Radial eq.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{dr^2}u_{nl} + \left[V(r) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u_{nl}(r) = E\,u_{nl}$$

$$\psi_{nlm}(r,\theta,\phi) = \frac{u_{nl}(r)}{r}\, Y_l^m(\theta,\phi)$$

B.C. $u \to 0$ as $r \to 0$
$r \to \infty$

For wave function to be normalizable

Review solution for H atom $V = \dfrac{-e^2}{4\pi\epsilon_0 r}$

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Angular momentum

$$L = \vec{r} \wedge \vec{p}$$

$$[L_x, L_y] = i\hbar L_z$$

but $[L_i, L^2] = 0$ $\quad i = x, y, z$

so can have simultanious eigenvalues for L^2, L_z

$$L^2 Y_l^m = \hbar^2 l(l+1) Y_l^m$$

$$L_z Y_l^m = \hbar m Y_l^m \qquad m = -l, -l+1 \ldots l+1, l$$

Addition of angular momentum

$$\vec{J} = \vec{L} + \vec{S}$$

J_z has eigenvalues $m_z = m + m_s$

J^2 has eigenvalues $\hbar^2 j(j+1)$

allowed $j = |l-s|, |l-s|+1 \ldots, l+s$
From triangle rule

Quantum Stat. Mechanics

a) $\frac{1}{2}$ integer spins are Fermions: Wave function must be antisymmetric under interchange of any two identical Fermions

b) Integer spins are bosons: wave function is sym.

c) Pauli exclusion principle

d) Maximize number of possible states for system in thermal equilibrium gives Fermi-Dirac dist.

$$n(E) = [e^{(E-\mu)/kT} + 1]^{-1}$$

or Bose-Einstien dist.

$$n(E) = [e^{(E-\mu)/kT} - 1]^{-1}$$

e) adjust μ to get correct # of particles.