4/23/01

Lecture 39 Perturbation Theory

Example: Relativistic corrections to kinetic energy

$$T = \sqrt{p^2c^2 + m^2c^4} - mc^2$$

$$= \frac{p^2}{2m} - \frac{p^4}{8m^3c^2} + \dots$$

$$= T_0 + H'$$

Correction $H' = -\frac{p^4}{8m^3c^2} = \frac{p^2}{2m}\left[\frac{\frac{p^2}{2m}}{2mc^2}\right]$

H' is very small. For a Hydrogen atom

$$\langle H' \rangle \sim \left\langle \frac{p^2}{2m} \right\rangle^2 \frac{1}{2mc^2}$$

$\left\langle \frac{p^2}{2m} \right\rangle = 13.6$ eV in ground state

Note $\langle V \rangle = -27.2$ eV

$$\left\langle \frac{p^2}{2m} \right\rangle + \langle V \rangle = -13.6 \text{ eV}$$

From Virial theorem $\langle V \rangle = 2E_n$

$$\langle H' \rangle \sim 13.6 \text{ eV} \; \frac{13.6 \text{ eV}}{511{,}000 \text{ eV}}$$

Correction of one part in $\sim 10^4$

$v/c \sim \alpha = \frac{1}{137}$

Correction order $v^2/c^2 \sim 10^{-4}$.

Therefore we expect true ground state wave function to be

$$\psi_{100} \simeq \psi^0_{100} + \psi'_{100}$$

$\psi^0_{100} = R_{10} Y^0_0$ old wave function

ψ'_{100} = very small correction of order one part in 10^4 of ψ^0_{100}