4/20/01

Lecture 30 Bose Einstein Condensation

See jilawww.colorado.edu/bec

First formed by Carl Weiman's group at Univ. of Colorado in 1995
Predicted by Bose + Einstein in 1924

Critical density ⇒ density of Bose gas as $\mu \to 0$

$$\rho_{crit} \sim \frac{1}{\lambda^3}$$

λ = thermal debroglie wavelength $= \frac{h}{p}$

$$\frac{p^2}{2m} \sim k_B T$$

$$\rho_{crit} \sim \left(\frac{2 m k_B T}{h^2}\right)^{3/2}$$

- Need either very high density or very low temperature because h is very small.

Assume ideal bose gas (neglect interactions between atoms)

$$\rho_{crit} = g \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{e^{E/k_BT} - 1} \right]$$

g = spin degeneracy = $2S+1$

$= 1 \, , \, 3$

$S = 0 \, , \, 1$

Bose gas S must be integer

$$E = \frac{\hbar^2 k^2}{2m} \qquad \text{let} \quad x = \frac{\hbar^2 k^2}{2m\,kT}$$

$$dx = \frac{\hbar^2}{2m\,kT}\, 2k\, dk$$

$$\boxed{\rho_{crit} = \frac{4\pi}{(2\pi)^3}\, g \left(\frac{2mkT}{\hbar^2}\right)^{3/2} \frac{1}{2} \int_0^\infty \frac{x^{1/2}\,dx}{e^x - 1}}$$

$$k = \left(\frac{2mkT}{\hbar^2}\right)^{1/2} x^{1/2}$$

$$\rho_{crit} = \frac{4\pi g}{(2\pi)^3} \int_0^\infty \frac{k^2\,dk}{e^{E_k/kT} - 1}$$

$$= \frac{g}{2\pi^2} \left(\frac{2mk_BT}{\hbar^2}\right)^{3/2} \frac{1}{2} \int_0^\infty \frac{x^{1/2}\,dx}{e^x - 1}$$

$$\int_0^\infty \frac{x^{s-1}\,dx}{e^x - 1} = \Gamma(s)\,\zeta(s)$$

[See 5.109]

$$\Gamma\left(\tfrac{3}{2}\right) = \frac{\pi^{1/2}}{2}$$

$$\zeta(s) = \sum_{p=1}^{\infty} p^{-s}$$

$$\zeta(3/2) = 2.612\ldots$$

$$\boxed{\rho_{crit} = \left[\frac{g\,\zeta(3/2)}{8\pi^{3/2}}\right]\left(\frac{2mk_BT}{\hbar^2}\right)^{3/2}}$$

Example Liquid ^4He at $T = 2$ K

^4He has $2p + 2n + 2e^-$

Can add an even # of spin $\frac{1}{2}$ particles to get an integer spin

$\Longrightarrow$ ^4He is a boson.

It has $S = 0 \Rightarrow g = 1$

$$m \simeq 4\,(1.661 \times 10^{-27}\ \mathrm{kg}) = 6.64 \times 10^{-27}\ \mathrm{kg}$$

$$k_B = 1.381 \times 10^{-23}\ \mathrm{J/K}$$

$$\hbar = 1.055 \times 10^{-34}\ \mathrm{J \cdot s}$$

$$\rho_{crit} = \left(\frac{2.612}{8\pi^{3/2}}\right)\left[\frac{2 \cdot 6.64 \times 10^{-27} \cdot 1.38 \times 10^{-23} \cdot 2}{(1.055 \times 10^{-34})^2}\right]^{3/2}$$

$$= .0586\,(1.89\,E29) = 1.107\,E28$$

$$\rho_{crit} = 1.11 \times 10^{28} \text{ particles}/m^3$$

$$= 1.11 \times 10^{22} \frac{\text{particles}}{cm^3}$$

$$= 1.11 \times 10^{22} \cdot 6.64 \times 10^{-24} g = 0.074 \text{ gm}/cm^3$$

This is close to the real density of superfluid ^4He $\rho = 0.15 \text{ gm}/cm^3$

Note ^4He is not a pure Bose condensate because interactions between ^4He atoms are very strong.

~~\#~~ Howeve transition to superfluid phase ~~to~~ at $T = 2.17 K$ is associated with the formation of a zero momentum condensate.

~~\#~~ If you cool superfluid ^4He to very low temperatures only about 10% of the atoms go into the condensate

To make a real Bose-Einstein condensate cool a very dilute gas to very low temperatures

If $\lambda >> r_i$

where r_i is the range of the interactions between atoms than system will behave as an ideal Bose gas.

In 1995 ^{87}Rb atoms were cooled in a magnetic trap.

Used several methods of cooling

Laser cooling

Use a laser with frequency just below an atomic transition

atom at rest or moving away does not absorb light

But an atom moving towards laser sees doppler shifted light beam and absorbs light, Momentum of light photon slows atom down.

$$p = \hbar\omega / c$$

Cooled system to $\sim$ 170 nanoK

$$T = 170 \times 10^{-9}\ K$$

^{87}Rb atoms have an odd # of electrons plus an odd # of neutrons + protons $\rightarrow$ sum to an integer spin (boson)

$Z = 37$ e^- + 37 protons + 50 neutrons

$$M \simeq 87\,(1.66 \times 10^{-27})\ kg = 1.44 \times 10^{-25}\ kg$$

$$k_B T = 1.38 \times 10^{-23}\ J/k \quad 170 \times 10^{-9} = 2.35 \times 10^{-30}\ J$$

$$\hbar = 1.05 \times 10^{-34}\ J/s$$

$$\rho_{crit} = 0.0586 \left[\frac{2 \quad 1.44 \times 10^{-25} \quad 2.35 \times 10^{-30}}{(1.05 \times 10^{-34})^2} \right]^{3/2}$$

$$= 2.8 \times 10^{19}\ \text{particles} / m^3$$

$$\boxed{\rho_{crit} = 2.8 \times 10^{13}\ \text{atoms} / cm^3}$$

Colorado group claimed slightly lower critical density but [illegible] measurement