Lecture 38  Bose Einstein Condensation

4/20/01

See jilawww.colorado.edu/bec

First formed by Carl Weiman's group at Univ. of Colorado in 1995
Predicted by Bose + Einstein in 1924

Critical density \( \equiv \) density of Bose gas as \( n \to 0 \)

\[
P_{\text{crit}} \sim \frac{1}{\lambda^3}
\]

\( \lambda = \text{thermal de Broglie wavelength} = \frac{\hbar}{p} \)

\[
P_{\text{crit}} \sim \left( \frac{2m kT}{\hbar^2} \right)^{3/2}
\]

- Need either very high density or very low temperature because \( \hbar \) is very small.

Assume ideal Bose gas (neglect interactions between atoms)
\[ P_{cr. t} = g \int \frac{d^3 k}{(2\pi)^3} \left( \frac{1}{e^{E/kT} - 1} \right) \]

\[ g = s p \text{ spin degeneracy } = 2s+1 \]

\[ s = 0, 1 \]

Bose gas \( s \) must be integer.

\[ E = \frac{\hbar^2}{2m} k^2 \]

\( \text{let} \quad x = \frac{\hbar^2 k^2}{2m kT} \)

\[ dx = \frac{k^2}{2m kT} 2k \, dk \]

\[ P_{cr. t} = \frac{4\pi g}{(2\pi)^3} \int \frac{e^{2x}}{e^{x} - 1} \]

\[ \int_0^{\infty} x^{s-1} e^{-x} \, dx = \Gamma(s) \Gamma(1) \]

\[ [\text{see 5.107}] \]
\[
S(3/2) = \frac{\pi^{1/2}}{2}
\]
\[
S(5) = \sum_{p=1}^{5} p
\]
\[
S(\frac{3}{2}) = 2.612...
\]

\[
P_{\text{crit}} = \left( \frac{g \cdot S(3/2)}{8 \pi^{3/2}} \right) \left( \frac{2 \cdot m \cdot k_{B} \cdot T}{\hbar^{2}} \right)^{3/2}
\]

Example: Liquid \( ^{4}\text{He} \) at \( T = 2 \text{ K} \)

\( ^{4}\text{He} \) has 2p + 2n + 2e\(^{-}\)

Can add an even \# of spin \( \frac{1}{2} \) particles to get an integer spin 

\( \Rightarrow ^{4}\text{He} \) is a boson.

It has \( S = 0 \Rightarrow g = 1 \)

\[M = 4 \times (1.66 \times 10^{-27} \text{ kg}) = 6.64 \times 10^{-27} \text{ kg}\]
\[k_{B} = 1.38 \times 10^{-23} \text{ J/} \text{K}\]
\[\hbar = 1.055 \times 10^{-34} \text{ J} \cdot \text{S}\]

\[
P_{\text{crit}} = \left( \frac{2.612}{8 \pi^{3/2}} \right) \left( \frac{2 \cdot 6.64 \times 10^{-27} \cdot 1.38 \times 10^{-23}}{1.055 \times 10^{-34}} \right)^{3/2}
\]

\[= 0.6586 \times (1.89 \times 29) = 1.167 \times 10^{3}\]
\[ P_{ei} = 1.11 \times 10^{28} \text{ particles/m}^3 \]
\[ = 1.11 \times 10^{22} \text{ particles/cm}^3 \]
\[ = 1.11 \times 10^{22} \times 6.64 \times 10^{-24} \text{ g} = 0.074 \text{ g/cm}^3 \]

This is close to the real density of superfluid $^4$He, \( \rho = 0.15 \text{ g/cm}^3 \).

Note: He is not a pure Bose condensate because interactions between the atoms are very strong.

However, transition to superfluid phase at \( T = 2.17 \text{ K} \) is associated with the formation of a zero momentum condensate.

To make a real Bose-Einstein condensate cool a very dilute gas to very low temperatures.
If $\lambda > \ell$,
where $\ell$ is the range of the interactions, between atoms, then system will behave as an ideal Bose gas.

In 1995 $^{87} \text{Rb}$ atoms were cooled in a magnetic trap. Used several methods of cooling.

Laser → <br>

Use a laser with frequency just below an atomic transition

But an atom moving towards laser sees doppler shifted light beam and absorb light. Momentum of light
\[ p = \frac{\hbar}{mc} \]

Cooled system to \( \sim 170 \text{ nanok} \)

\[ T = 170 \times 10^{-9} \text{ K} \]

\( ^{87}\text{Rb} \) atoms have an odd # of electrons plus an odd # of neutrons \( \Rightarrow \) sum to an integer spin \((\text{boson})\)

\[ Z = 37 \text{ e}^- + 37 \text{ protons} + 50 \text{ neutrons} \]

\[ M = 87 \left(1.66 \times 10^{-27}\right) \text{ kg} = 1.44 \times 10^{-25} \text{ kg} \]

\[ k_B T = 1.38 \times 10^{-23} \text{ J/k} \quad 170 \times 10^{-9} = 2.35 \times 10^{-5} \text{ J} \]

\[ T = 1.65 \times 10^{-34} \text{ J/K} \]

\[ P_{\text{crit}} = 0.05 \text{ Pa} \]

\[ \sqrt{\frac{2 \times 1.44 \times 10^{-25}}{1.05 \times 10^{-24}} \times \frac{2.35 \times 10^{-36}}{2}} \]

\[ \rho_{\text{crit}} = 2.8 \times 10^{13} \text{ atoms/m}^3 \]

Colorado group claimed slightly lower critical density but still on point in \( \hbar \). Measurement.