Virtual Photons in Chiral Perturbation Theory

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Workshop on Charge Symmetry Breaking and Other Isospin Violations

Trento, 13/6/2005
• Why electromagnetic corrections for strong processes?
• Chiral Perturbation Theory with photons
• Quark mass ratios and Dashen’s theorem
• $\pi\pi$, $\pi K$ scattering at threshold
• Summary / Beyond ...
Why electromagnetic corrections for strong processes?

Processes / observables forbidden by isospin

- two sources of isospin breaking:
  
  "strong" $m_u - m_d$ + "electromagnetic" $\alpha_{\text{QED}}$

- three qualitative cases:

  1. $m_u - m_d$ dominates; example: $\eta \rightarrow 3\pi$
     (Sutherland theorem: no electromagnetic effects in chiral limit)

  2. electromagnetism dominates; example: $M_{\pi^+}^2 - M_{\pi^0}^2$,
     strong effects suppressed to $O((m_u - m_d)^2)$

  3. both of similar importance; example: $M_{K^+}^2 - M_{K^0}^2$
High-precision isospin-allowed processes
as soon as hadronic processes are measured/predicted at the few % level
⇒ consider radiative (and other isospin breaking) corrections

Examples:
  • $K\ell\ell$ decays, extraction of $V_{us}$ \hspace{1cm} H. Neufeld’s talk
  • $K \rightarrow 3\pi$ \hspace{1cm} J. Bijnens’s talk
  • $\pi\pi$ ($\pi K$) scattering at threshold ⇒ pionium ($\pi K$ atoms)
Chiral Perturbation Theory with photons

Leading order chiral Lagrangian:

\[
\mathcal{L}^{(2)} = \frac{F^2}{4} \left\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \right\rangle
\]

\[
D_\mu U = \partial_\mu U - i [v_\mu, U] - i \{a_\mu, U\}
\]

⇒ coupling to external photons straightforward with \( v_\mu = e A_\mu \):

(pion vector form factor, Compton scattering)
Simply include photons via minimal substitution and close the loop?

\[ (M_{\pi^+}^2)_{\text{em}} \propto e^2 M_{\pi^+}^2 \rightarrow 0 \quad \text{in the chiral limit} \]

model using vector ($\rho^0$) and axial vector ($a_1$) exchange

finite using Weinberg sum rule

\[ M_{\rho}^2 F_{\rho}^2 = M_{a_1}^2 F_{a_1}^2 \]

\[ (M_{\pi^+}^2)_{\text{em}} = \frac{3 e^2}{16 \pi^2} \frac{F_{\rho}^2 M_{\rho}^2}{F_{\pi}^2} \log \frac{M_{a_1}^2}{M_{\rho}^2} \]

\[ \rightarrow \text{finite in the chiral limit} \quad \text{T. Das et al. 1967} \]

\[ \Rightarrow \text{Inclusion of virtual photons via minimal substitution only} \]

\[ \text{does not generate the most general electromagnetic effects!} \]
Construction principle: spurion formalism

- Reminder:
  transformation law for quark mass term under chiral group

\[ \mathcal{L}_{\text{mass}}^{QCD} = -\bar{q} \mathcal{M} q \rightarrow -\bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M}^\dagger q_L \]

\Rightarrow \text{introduce scalar source with transformation law } s \rightarrow g_L s g_R^\dagger \text{ set } s = \mathcal{M} = \text{diag}(m_u, m_d, m_s) \text{ in the end}

- now: transformation law for quark charge term

\[ \mathcal{L}_{\text{em}}^{QCD} = -\bar{q} \mathcal{A} q \rightarrow -\bar{q}_L \mathcal{A} q_L - \bar{q}_R \mathcal{A} q_R \]

\Rightarrow \text{deduce transformation laws } Q_L \rightarrow g_L Q_L g_L^\dagger, \ Q_R \rightarrow g_R Q_R g_R^\dagger, \text{ set } Q_L = Q_R = Q = e \text{ diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}) \text{ in the end}

- generalised power counting: \( Q_{L/R} = \mathcal{O}(p), \ A_\mu = \mathcal{O}(1) \)
Electromagnetism at lowest order

- only term in the chiral Lagrangian at $\mathcal{O}(e^2) = \mathcal{O}(p^2)$:

\[
\mathcal{L}_{\text{em}}^{(2)} = C \langle Q_L U Q_R U^\dagger \rangle
\]

- contributions to the meson masses at leading order:

\[
M_{\pi^+}^2 = B(m_u + m_d) + \frac{2Ce^2}{F^2} \quad M_{\pi^0}^2 = B(m_u + m_d)
\]

\[
M_{K^+}^2 = B(m_u + m_s) + \frac{2Ce^2}{F^2} \quad M_{K^0}^2 = B(m_d + m_s)
\]

\[
M_{\eta}^2 = \frac{B}{3}(m_u + m_d + 4m_s)
\]

$\Rightarrow$ Dashen’s theorem:

\[
(M_{\pi^+}^2 - M_{\pi^0}^2)_{\text{em}} = (M_{K^+}^2 - M_{K^0}^2)_{\text{em}}
\]
in the chiral limit!

- no contributions to neutral masses, $\pi^0 \eta$ mixing, $\eta \rightarrow 3\pi$
the charged-to-neutral pion mass difference is predominantly electromagnetic:

\[
M_{\pi^+} - M_{\pi^0} = \frac{Ce^2}{M_\pi F_\pi^2} + \mathcal{O}((m_u - m_d)^2)
\]

\[\approx 4.6 \text{ MeV}\]

\[\approx 0.1 \text{ MeV}\]

\[\Rightarrow \text{ fix electromagnetic low-energy constant } C\]

\[
C = \frac{F_\pi^2}{2e^2} (M_{\pi^+}^2 - M_{\pi^0}^2)
\]

• sum rule

\[
\frac{1}{2M_\pi} (M_{\pi^+}^2 - M_{\pi^0}^2) = \frac{3e^2}{32\pi^2} \frac{F_\rho^2 M_\rho^2}{M_\pi F_\pi^2} \log \frac{M_{a_1}^2}{M_\rho^2} \approx 5.0 \text{ MeV}
\]

turns out to yield very good estimate of \(C\) \[T. \text{Das et al. 1967}\]
Quark mass ratios and Dashen’s theorem

• consequence for the double ratio of quark masses

\[ Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} = \frac{M_K^2}{M_\pi^2} \cdot \frac{M_K^2 - M_\pi^2}{(M_{K^0}^2 - M_{K^+}^2)_{QCD}} \cdot \left\{ 1 + O(m_q^2) \right\} \]

(major semi-axis of the Leutwyler ellipse) G. Colangelo, this morning

• using Dashen’s theorem to evaluate \((M_{K^0}^2 - M_{K^+}^2)_{QCD}\) leads to

\[ Q^2_D = 24.2 \]

• for comparison: \(\Gamma(\eta \rightarrow 3\pi) \propto 1/Q^4\) leads to

\[ Q^2 = 22.4 \pm 0.9 \quad \text{J. Kambor, C. Wiesendanger, D. Wyler 1995} \]

\[ Q^2 = 22.8 \pm 0.4 \quad \text{B.V. Martemyanov, V.S. Sopov 2005} \]

⇒ large corrections to Dashen’s theorem?

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Electromagnetism at next-to-leading order

- $\mathcal{O}(e^2 p^2)$: photon loops, meson loops with mass differences,
  + 14 new counterterms: R. Urech 1995

$$
\mathcal{L}_{\em}^{(4)} = \frac{1}{2} K_1 F^2 \langle D_\mu U D^\mu U^\dagger \rangle \langle Q_L Q_L + Q_R Q_R \rangle + \ldots 
+ \frac{1}{2} K_7 F^2 \langle \chi U^\dagger \chi^\dagger U \rangle \langle Q_L Q_L + Q_R Q_R \rangle + \ldots
$$

- some subtleties:
  - spurion technique → anomaly-free theory?
  - $K_i$ in general gauge-dependent
  - separation strong ↔ electromagnetic ambiguous
    $K_i$ depend also on the QCD scale B. Moussallam 1997
    J. Gasser, A. Rusetsky, I. Scimemi 2003
Corrections to Dashen’s theorem at $\mathcal{O}(m_q e^2)$

\[
(M_{K^+}^2 - M_{K^0}^2)_{em} - (M_{\pi^+}^2 - M_{\pi^0}^2)_{em} = \text{(photon loops)} \propto e^2(M_K^2 - M_\pi^2) \\
+ \text{(mass differences in strong loops)} \propto C e^2\{M_{K,\pi}^2\} \\
+ \text{(mass differences in strong counterterms)} \propto C e^2 L_5^r (M_K^2 - M_\pi^2) \\
+ K_{\pi}^r M_\pi^2 + K_{K}^r M_K^2 \text{ (electromagnetic counterterms)} \\
+ \mathcal{O}(e^2 m_q)
\]

$\Rightarrow$ need estimates for electromagnetic low-energy constants $K_i$

in order to quantify corrections to Dashen’s theorem
Estimates of the electromagnetic low-energy constants $K_i$

- **Resonance saturation**:
  - construct Lagrangian coupling resonances to Goldstone bosons
  - decomposition of low-energy constants
    \[
    L_{ri}(\mu) = \sum_{R=V,A,S,P} L_{ri}^R + \hat{L}_{ri}(\mu) \]
    - observation: $\hat{L}_{i}(M_{\rho}) \approx 0$ (modern “vector meson dominance”)
      
      G. Ecker, J. Gasser, A. Pich, E. de Rafael 1988

- electromagnetic low-energy constants: resonances in loops
  - photon loops divergent $\Rightarrow$ QCD scale dependence of $K_{i}^{R}$
  - assume saturation, no sufficient phenomenological evidence

   R. Bauer, R. Urech 1997; B. Moussallam 1997
Estimates of the electromagnetic low-energy constants $K_i$ (2)

R. Bauer, R. Urech 1997

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Corrections to Dashen’s theorem, quark mass ratio $Q^2$

<table>
<thead>
<tr>
<th>method</th>
<th>$\Delta M^2_K / \Delta M^2_\pi$</th>
<th>$Q^2$</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>resonance saturation</td>
<td>1</td>
<td>24.2</td>
<td>Dashen 1969</td>
</tr>
<tr>
<td>1/$N_c$, ENJL+PQCD</td>
<td>1.1 ± 0.2</td>
<td>24.0 ± 0.6</td>
<td>Baur, Urech 1996</td>
</tr>
<tr>
<td>Cottingham method</td>
<td>1.8 ± 0.2</td>
<td>22.0 ± 0.6</td>
<td>Bijnens, Prades 1997</td>
</tr>
<tr>
<td>sum rules</td>
<td>2.0 ± 0.4</td>
<td>21.6 ± 0.9</td>
<td>Donoghue, Pérez 1997</td>
</tr>
<tr>
<td>Lattice</td>
<td>2.5</td>
<td>20.6</td>
<td>Ananthanarayan, Moussallam 2004</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>22.9</td>
<td>Duncan et al. 1996</td>
</tr>
</tbody>
</table>

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Corrections to Dashen’s theorem, quark mass ratio $Q^2$ (2)

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inspired by H. Leutwyler 1996
**Hadronic atoms**

- DIRAC experiment at CERN measures lifetimes and $2S - 2P$ energy level shifts in $\pi^+\pi^-$, $\pi^+K^-$ atoms

- first result: $\tau_{\pi^+\pi^-} = (2.91^{+0.49}_{-0.62}) \times 10^{-15}$ s

- sensitive to $\pi\pi$, $\pi K$ scattering amplitudes at threshold:

$$\Gamma_{n0} = \frac{8\alpha^3\mu_+^2}{n^3} A^2 (1 + K) \ , \ \mathcal{A} \propto \text{Re} A_{\text{thr}}^{\pm;00}$$

$$\Delta E_{n0}^{\text{strong}} = \frac{2\alpha^3\mu_+^2}{n^3} \mathcal{A}' (1 + K') \ , \ \mathcal{A}' \propto \text{Re} A_{\text{thr}}^{\pm;\pm}$$

$\Rightarrow$ modified Deser formulae

- amplitudes at threshold $\propto$ scattering lengths $+$ corrections!

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\(\pi\pi\) scattering length from \(K \rightarrow 3\pi\)

- \(K^+ \rightarrow \pi^+\pi^0\pi^0\): intermediate \(\pi^+\pi^-\) state induces cusp in \(\pi^0\pi^0\) invariant mass spectrum

\[
\begin{align*}
\pi^+ & \quad \pi^+ \\
\pi^- & \quad \pi^0 \\
K^+ & \quad \pi^0 \\
\end{align*}
\]

N. Cabibbo 2004; N. Cabibbo, G. Isidori 2005

- measure cusp strength to few % accuracy: NA48 2005
\[
\propto A(\pi^+\pi^- \rightarrow \pi^0\pi^0) \propto a_0^2 - a_0^0 + \text{isospin-breaking corrections}
\]

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Scattering lengths at tree level

- express all electromagnetic corrections in terms of $\Delta M_{\pi}^2$, all $m_u - m_d$ effects in terms of $\epsilon = \epsilon_{\pi^0 \eta}$

- charge exchange reactions ($\rightarrow$ lifetime):

$$a_0(\pi^- \pi^+ \rightarrow \pi^0 \pi^0) = \frac{1}{3} \left( a_0^2 - a_0^0 \right) \left\{ 1 + \frac{\Delta M_{\pi}^2}{3M_{\pi}^2} \right\}$$

$$a_0(\pi^- K^+ \rightarrow \pi^0 K^0) = \frac{\sqrt{2}}{3} \left( a_0^{3/2} - a_0^{1/2} \right) \left\{ 1 + \frac{\epsilon}{\sqrt{3}} + \frac{\Delta M_{\pi}^2}{2M_{\pi}M_K} \right\}$$

⇒ isospin breaking corrections are

- electromagnetic only & sizeable for $\pi\pi$ scattering
- both electromagnetic and strong & smaller for $\pi K$ scattering

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Scattering lengths at tree level (2)

• elastic charged reactions ($\rightarrow$ strong energy level shift):

\[ a_0(\pi^-\pi^+ \rightarrow \pi^-\pi^+) = \frac{1}{6} \left( 2a_0^0 + a_0^2 \right) \left\{ 1 + \frac{\Delta M_{\pi}^2}{M_{\pi}^2} \right\} \]

\[ a_0(\pi^-K^+ \rightarrow \pi^-K^+) = \frac{1}{3} \left( 2a_0^{1/2} + a_0^{3/2} \right) \left\{ 1 + \frac{\Delta M_{\pi}^2}{M_{\pi}M_K} \right\} \]

\[ \Rightarrow \text{isospin breaking corrections are} \]

– electromagnetic only in both cases

– smaller for $\pi K$ due to bigger $M_K$
Scattering lengths at one-loop level

• matching condition for hadronic atoms: subtract Coulomb pole

$\Rightarrow \text{Re } \mathcal{A} = \frac{e^2 \pi M_\pi M_K}{q} a_0^{\text{tree}} + 8\pi (M_\pi + M_K) a_0^{\text{tree}+\text{loop}} + \mathcal{O}(q)$

J. Gasser et al. 2001
Scattering lengths at one-loop level (2)

\[ a_0(\pi^- \pi^+ \rightarrow \pi^0 \pi^0) = \frac{1}{3} \left( a_0^2 - a_0^0 \right) \left\{ 1 + \left( 2.3 \pm 0.6 \right)\% \right\} \]

Instead of 2.1%  

M. Knecht, R. Urech 1998

\[ a_0(\pi^- K^+ \rightarrow \pi^0 K^0) = \frac{\sqrt{2}}{3} \left( a_0^{3/2} - a_0^{1/2} \right) \left\{ 1 + \left( 1.3 \pm 1.3 \right)\% \right\} \]

Instead of 1.9%  


\[ a_0(\pi^- \pi^+ \rightarrow \pi^- \pi^+) = \frac{1}{6} \left( 2a_0^0 + a_0^2 \right) \left\{ 1 + \left( 5.6 \pm 1.2 \right)\% \right\} \]

Instead of 6.4%  

M. Knecht, A. Nehme 2002

\[ a_0(\pi^- K^+ \rightarrow \pi^- K^+) = \frac{1}{3} \left( 2a_0^{1/2} + a_0^{3/2} \right) \left\{ 1 + \left( 1.1 \pm 3.2 \right)\% \right\} \]

Instead of 1.8%  


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Summary

• Chiral Perturbation Theory has to be amended with new operators in order to accommodate the most general electromagnetic effects.

• Lagrangians $\mathcal{L}_{em}^{(2)}$ and $\mathcal{L}_{em}^{(4)}$ known estimating the low-energy constants $K_i$ a challenge.

• Potentially large corrections to Dashen’s theorem reduce the double quark mass ratio $Q^2 = (m_s^2 - \hat{m}^2)/(m_d^2 - m_u^2)$ $\Rightarrow$ reconciliation with $\eta \rightarrow 3\pi$?

• Need isospin breaking corrections for precise extraction of $\pi\pi$, $\pi K$ scattering lengths from hadronic atoms or $K \rightarrow 3\pi$. 

Virtual Photons in Chiral Perturbation Theory
Beyond . . .

- many other applications, in particular meson decays
  
  talks by M. Knecht, H. Neufeld, J. Bijnens . . .

- semi-leptonic decays require yet another generalisation:
  
  include virtual leptons
  
  M. Knecht, H. Neufeld, H. Rupertsberger, P. Talavera 1999

- virtual photons in baryon ChPT:
  
  
  isospin violation in $\pi N$ scattering, pionic hydrogen
  
  N. Fettes, U.-G. Meißner 2001; J. Gasser et al. 2002; talk by D. Gotta