The masses of the light quarks

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Outline

Introduction

Quark mass ratios

Sum rules

Lattice
Quantum Chromodynamics

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} \text{Tr} G_{\mu\nu} G^{\mu\nu} + \sum_i \bar{q}_i (i\slashed{D} - m_{q_i}) q_i \]

- QCD is a confining theory \( \Rightarrow \)
  quark masses cannot be measured as lepton masses
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  - if \( m_{q_i} / \Lambda_{\text{QCD}} \ll 1 \) \( \Rightarrow \) expand around \( m_{q_i} / \Lambda_{\text{QCD}} = 0 \)
Quantum Chromodynamics

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- if \( m_{q_i}/\Lambda_{\text{QCD}} \ll 1 \) ⇒ expand around \( m_{q_i}/\Lambda_{\text{QCD}} = 0 \)
- if \( m_{q_i}/\Lambda_{\text{QCD}} \gg 1 \) ⇒ expand around \( \Lambda_{\text{QCD}}/m_{q_i} = 0 \)
QCD in the chiral limit

QCD Lagrangian with quark masses set to zero:

\[ \mathcal{L}_{QCD}^{(0)} = -\frac{1}{4} \text{Tr} G_{\mu\nu} G^{\mu\nu} + \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R \]

\[ q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad q_{R,L} = \frac{1}{2} (1 \pm \gamma_5) q \]

Large global symmetry group:

\[ SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A \]

Spontaneous symmetry breaking

\[ SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \]

\[ \Rightarrow \text{octet of Goldstone bosons} = \pi, K, \eta \]
Expansion around the chiral limit

\[ \mathcal{H}_{\text{QCD}} = \mathcal{H}_{\text{QCD}}^0 + \mathcal{H}_m \]

\[ \mathcal{H}_m := \bar{q} \mathcal{M} q \]

\[ \mathcal{M} = \begin{pmatrix} m_u & m_d \\ m_d & m_s \end{pmatrix} \]

the mass term \( \mathcal{H}_m \) will be treated as a small perturbation
Expansion around the chiral limit

\[ \mathcal{H}_{\text{QCD}} = \mathcal{H}_{\text{QCD}}^0 + \mathcal{H}_m \quad \mathcal{H}_m := \bar{q} M q \quad M = \begin{pmatrix} m_u & m_d \\ m_d & m_s \end{pmatrix} \]

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Expansion around \( \mathcal{H}_{\text{QCD}}^0 \equiv \text{expansion in powers of } m_q \)
Expansion around the chiral limit

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Expansion around \( \mathcal{H}_{\text{QCD}}^0 \) \( \equiv \) expansion in powers of \( m_q \)

General quark mass expansion for a particle \( P \):

\[ M^2 = M_0^2 + (m_u + m_d) \langle P | \bar{q} q | P \rangle + O(m_q^2) \]
Expansion around the chiral limit

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\[ M^2 = M_0^2 + (m_u + m_d) \langle P | \bar{q} q | P \rangle + O(m_q^2) \]

For a Goldstone boson \( M_0^2 = 0 \):

\[ M_\pi^2 = -(m_u + m_d) \frac{1}{F_\pi^2} \langle 0 | \bar{q} q | 0 \rangle + O(m_q^2) \]

where we have used a Ward identity:

Gell-Mann, Oakes and Renner (68)

\[ \langle \pi | \bar{q} q | \pi \rangle = - \frac{1}{F_\pi^2} \langle 0 | \bar{q} q | 0 \rangle =: B_0 \]
Quark masses

Consider the whole pseudoscalar octet:

\[ M_{\pi}^2 = B_0(m_u + m_d) \quad M_{K^+}^2 = B_0(m_u + m_s) \quad M_{K^0}^2 = B_0(m_d + m_s) \]
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Quark mass ratios:

\[
\frac{m_u}{m_d} \approx \frac{M_{\pi^+}^2 - M_{K^0}^2 + M_{K^+}^2}{M_{\pi^+}^2 + M_{K^0}^2 - M_{K^+}^2} \approx 0.67
\]

\[
\frac{m_s}{m_d} \approx \frac{M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} \approx 20
\]
Quark masses

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\]

\[ \hat{m} \equiv (m_u + m_d)/2 \approx 5.4 \text{ MeV} \]

SU(6) relation, Leutwyler (75)

\[ m_u \approx 4 \text{ MeV} \quad m_d \approx 6 \text{ MeV} \quad m_s \approx 135 \text{ MeV} \]

Gasser and Leutwyler (75)
Electromagnetic corrections to the masses

According to Dashen’s theorem

\[
\begin{align*}
    M_{\pi^0}^2 &= B_0(m_u + m_d) \\
    M_{\pi^+}^2 &= B_0(m_u + m_d) + \Delta_{\text{em}} \\
    M_{K^0}^2 &= B_0(m_d + m_s) \\
    M_{K^+}^2 &= B_0(m_u + m_s) + \Delta_{\text{em}}
\end{align*}
\]

Extracting the quark mass ratios gives

\[
\begin{align*}
    \frac{m_u}{m_d} &= \frac{M_{K^+}^2 - M_{K^0}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} \quad = 0.56 \\
    \frac{m_s}{m_d} &= \frac{M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} \quad = 20.1
\end{align*}
\]

Weinberg (77) estimated \( m_s \) from the splitting in baryon octet

\[
    m_u = 4.2 \text{ MeV} \quad m_d = 7.5 \text{ MeV} \quad m_s = 150 \text{ MeV}
\]
Higher order chiral corrections

Mass formulae to second order

\[
\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{2\hat{m}} \left[ 1 + \Delta_M + \mathcal{O}(m^2) \right]
\]

\[
\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2} = \frac{m_d - m_u}{m_s - \hat{m}} \left[ 1 + \Delta_M + \mathcal{O}(m^2) \right]
\]

\[
\Delta_M = \frac{8(M_K^2 - M_\pi^2)}{F_\pi^2} (2L_8 - L_5) + \chi\text{-logs}
\]

The same \( \mathcal{O}(m) \) correction appears in both ratios
\[\Rightarrow \text{this double ratio is free from } \mathcal{O}(m) \text{ corrections} \]

\[
Q^2 \equiv \frac{M_K^2}{M_\pi^2} \frac{M_{K^0}^2 - M_{K^+}^2}{M_{K^0}^2 - M_{K^+}^2} = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \left[ 1 + \mathcal{O}(m^2) \right]
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The same \( \mathcal{O}(m) \) correction appears in both ratios

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\[
Q_D^2 \equiv \frac{(M_{K^0}^2 + M_{K^+}^2 - M_{\pi^0}^2 + M_{\pi^+}^2)(M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2 - M_{\pi^0}^2)}{4M_{\pi^0}^2(M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2 - M_{\pi^0}^2)} = 24.2
\]
Leutwyler’s ellipse

Leutwyler observed that the information on $Q$ amounts to an elliptic constraint in the plane of the two mass ratios $\frac{m_s}{m_d}$ and $\frac{m_u}{m_d}$

\[
\left( \frac{m_s}{m_d} \right) \frac{1}{Q^2} + \left( \frac{m_u}{m_d} \right) = 1
\]
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$$\left(\frac{m_s}{m_d}\right) \frac{1}{Q^2} + \left(\frac{m_u}{m_d}\right) = 1$$

Weinberg (77)
Estimate of $Q$: violation of Dashen’s theorem

According to Dashen

$\left( M_{K^+}^2 - M_{K^0}^2 \right)_{em} = \left( M_{\pi^+}^2 - M_{\pi^0}^2 \right)_{em} \Rightarrow (M_{K^+} - M_{K^0})_{em} = 1.3$ MeV
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Higher order corrections change the numerical value

$$ (M_{K^+} - M_{K^0})_{em} = \begin{cases} 1.9 \text{ MeV} & \text{Duncan et al. (96)} \qquad Q = 22.8 \\ 2.3 \text{ MeV} & \text{Bijnens-Prades (97)} \qquad Q = 22 \\ 2.6 \text{ MeV} & \text{Donoghue-Perez (97)} \qquad Q = 21.5 \end{cases} $$

The last two model calculations agree within the uncertainty estimate of Bijnens-Prades

$$ Q = 22.0 \pm 0.6 $$
Estimate of $Q$: the decay $\eta \rightarrow \pi^0 \pi^+ \pi^-$

Leading order formula

$$A(\eta \rightarrow \pi^0 \pi^+ \pi^-) = -\frac{\sqrt{3}}{4} \frac{m_u - m_d}{m_s - \hat{m}} \frac{s - 4M_{\pi}^2/3}{F_{\pi}^2}$$
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The decay width can be written as

$$\Gamma(\eta \rightarrow \pi^0 \pi^+ \pi^-) = \Gamma_0 \left( \frac{Q_D}{Q} \right)^4 = (292 \pm 17) \text{ eV} \quad \text{PDG (04)}$$
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$$\Gamma_0 = \begin{cases} 
(167 \pm 50) \text{ eV} & \text{Gasser-Leutwyler (85)} \quad Q = 21.1 \pm 1.6 \\
(219 \pm 22) \text{ eV} & \text{Anisovich-Leutwyler (96)} \quad Q = 22.6 \pm 0.7 \\
(209 \pm 20) \text{ eV} & \text{Kambor et al (85)} \quad Q = 22.3 \pm 0.6 
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Confirmed by a recent Dalitz-plot analysis of KLOE data on $\eta \rightarrow 3\pi$

$$Q = 22.8 \pm 0.4$$

Martemyanov-Sopov (05)
Estimates of $Q$: summary

\[ Q_D = 24.2 \]
\[ Q = 22.6 \pm 0.8 \]
Kaplan-Manohar ambiguity

Phenomenological information alone does not allow one to fix both mass ratios

Kaplan-Manohar (86)
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- The quark mass matrix $\mathcal{M}$ transforms like $(\mathcal{M}^+)^{-1} \det(\mathcal{M})$
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Phenomenological information alone does not allow one to fix both mass ratios

- The quark mass matrix $\mathcal{M}$ transforms like $(\mathcal{M}^+)^{-1} \det(\mathcal{M})$
- The chiral Lagrangian with only $v$ and $a$ external fields is invariant under

$$m'_u = \alpha_1 m_u + \alpha_2 m_d m_s \quad \text{(and } u \rightarrow d \rightarrow s)$$

$$B' = B/\alpha_1$$

$$L'_6 = L_6 - \alpha$$

$$L'_7 = L_7 - \alpha$$

$$L'_8 = L_8 + \alpha$$

$$\alpha = \frac{\alpha_2 F^2}{32 \alpha_1 B}$$

but $Q' = Q$ !
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L_6' &= L_6 - \alpha \\
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L_8' &= L_8 + \alpha
\end{align*}
\]

\[
\alpha = \frac{\alpha_2 F^2}{32 \alpha_1 B}
\]

but $Q' = Q$ !

- Phenomenology alone cannot exclude $m_u = 0$ which would solve the strong CP-problem

Kaplan-Manohar (86)
The weird world with $m_u = 0$

- The KM ambiguity is not a symmetry of QCD

$$(m_u + m_d) \left< 0 | \overline{d} i \gamma_5 u | \pi^+ \right> = \sqrt{2} F_\pi M_{\pi^+}^2$$

This relation is not invariant under a KM transformation

Lattice QCD calculations can settle the issue
The weird world with $m_u = 0$ mostly Leutwyler’s arguments

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- If $m_u = 0$ a LO $\chi$PT prediction would be wrong by a factor 4

$$M_{K^0}^2 - M_{K^+}^2 = B_0 m_d - \Delta M_{em}^2 \quad \text{if} \quad m_u = 0 \quad 2M_{\pi^0}^2 - M_{\pi^+}^2$$

$$4.0 \cdot 10^{-3} \text{GeV}^2 = 17 \cdot 10^{-3} \text{GeV}^2$$
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- There would be very strong flavour violations

\[
\langle K^+|\bar{u}u - \bar{d}d|K^+\rangle \equiv S_{K^+}(t) \quad \langle \pi^+|\bar{u}u - \bar{s}s|\pi^+\rangle \equiv S_{\pi^+}(t)
\]

\[
S_{K^+}(0) = \left(\frac{\partial}{\partial m_u} - \frac{\partial}{\partial m_d}\right)M_{K^+}^2 \quad S_{\pi^+}(0) = \left(\frac{\partial}{\partial m_u} - \frac{\partial}{\partial m_s}\right)M_{\pi^+}^2
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$$ \langle K^+ | \bar{u}u - \bar{d}d | K^+ \rangle \equiv S_{K^+}(t) \quad \langle \pi^+ | \bar{u}u - \bar{s}s | \pi^+ \rangle \equiv S_{\pi^+}(t) $$

$$ r = \frac{S_{K^+}(0)}{S_{\pi^+}(0)} = \left[ \frac{m_s - m_u}{m_d - m_u} \frac{M_{K_0}^2 - M_{K^+}^2}{M_{K_0}^2 - M_{\pi^+}^2} \right]^2 (1 + O(m^2)) \quad \text{if } m_u = 0 \quad \approx 0.3 $$
\[ \Pi_{\mu\nu}^I(p) \equiv i \int dx e^{ipx} \langle 0 | TJ^I_{\mu}(x) J^{I\dagger}_\nu(0) | 0 \rangle \quad I = V, A \]
\[ = (p_\mu p_\nu - g_{\mu\nu} p^2) \Pi^{T,I}(p^2) + p_\mu p_\nu \Pi^{L,I}(p^2) \]

\[ R_\tau = \frac{\Gamma(\tau \rightarrow \text{hadrons } \nu_\tau)}{\Gamma(\tau \rightarrow e\nu_e \nu_\tau)} = R_{\tau,V+A} + R_{\tau,S} \]
\[ = 12\pi \int_0^1 dx (1-x)^2 \left[ (1+2x) \text{Im} \Pi^{T} + \text{Im} \Pi^{L} \right] \quad \left[ x = \frac{S}{M^2_\tau} \right] \]

\[ \Pi^J = |V_{ud}|^2 \left[ \Pi^{V,J} + \Pi^{A,J} \right] + |V_{us}|^2 \Pi^J_S \]

\[ R_{\tau}^{kl} \equiv \int_0^1 dx (1-x)^k x^l \frac{dR_\tau}{dx} \quad \delta R_{\tau}^{kl} = \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \]
\( R_{\tau}^{kl} \equiv \int_0^1 dx (1 - x)^k x^l \frac{dR_{\tau}}{dx} \)

\[ \delta R_{\tau}^{kl} = \frac{R_{\tau, V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau, S}^{kl}}{|V_{us}|^2} \]

These \( \delta R_{\tau}^{kl} \) are:

- zero in the \( SU(3) \) limit, i.e. sensitive to \( m_s \)
- calculable in perturbative QCD (in terms of \( m_s \))
- measurable (provided one knows \( V_{ud,s} \))

Experiment = QCD \( \Rightarrow m_s \)
\[ m_S \text{ from } \tau \text{ decays} \]

\[ \Delta_{kl}^{L+T} m_S^2 = \frac{M_{\tau}^2}{18(1 - \epsilon_d^2)} \left[ \frac{\delta R_{\tau}^{kl}}{S_{EW}} - \delta R_{\tau,D \geq 4}^{kl,L+T} \right] \]

\[ \epsilon_d = \frac{m_d}{m_s} \quad \delta R_{\tau,D \geq 4}^{kl,L+T} \text{ are higher dim. operators} \]
### Summary of recent results

\[
m_s(2\text{GeV}) \text{ (MeV)}
\]

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Jamin et al. (02)</td>
<td>99 ± 16</td>
</tr>
<tr>
<td>Kambor Maltman (02)</td>
<td>100 ± 12</td>
</tr>
<tr>
<td>Gamiz et al. (03)</td>
<td>103 ± 17</td>
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<td></td>
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<tr>
<td>Jamin et al. (05)</td>
<td>81 ± 22</td>
</tr>
<tr>
<td>Gorbunov-Pivovarov (05)</td>
<td>125 ± 28</td>
</tr>
<tr>
<td>Baikov et al. (05)</td>
<td>96 ± 19</td>
</tr>
<tr>
<td>Narison (05)</td>
<td>89 ± 25</td>
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</tbody>
</table>

The first set is mostly based on ALEPH
The second one also on newer data from CLEO and OPAL
Lattice determinations of $m_u$, $m_d$ and $m_s$

During last year there have been lattice calculations with 3 dynamical quarks and small quark masses (MILC: $\hat{m} \sim m_s/8$)
Lattice determinations of $m_u$, $m_d$ and $m_s$

During last year there have been lattice calculations with 3 dynamical quarks and small quark masses (MILC: $\hat{m} \sim m_s/8$)

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>$m_s(2\text{GeV})$ (MeV)</th>
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<tbody>
<tr>
<td>HPQCD–MILC–UKQCD (04)</td>
<td>76 ± 7</td>
</tr>
<tr>
<td>staggered fermions</td>
<td></td>
</tr>
<tr>
<td>CP-PACS–JLQCD (04)</td>
<td>(K) 80.4 ± 1.9</td>
</tr>
<tr>
<td>Wilson improved</td>
<td>(\phi) 89.3 ± 2.9</td>
</tr>
<tr>
<td>QCDSF–UKQCD (04)</td>
<td>119 ± 9</td>
</tr>
<tr>
<td>Wilson improved</td>
<td></td>
</tr>
</tbody>
</table>
Lattice determinations of $m_u$, $m_d$ and $m_s$

![Graph showing the lattice determinations of $m_u$, $m_d$, and $m_s$. The graph includes lines and shaded areas with labels indicating the ratios and uncertainties.]

- $m_s/m_d = 27.4 \pm 0.5$
- $m_s/m_d = 26 \pm 1$
- $Q_D = 24.2$
- Weinberg (77)
- $Q = 22.6 \pm 0.8$
The ratios of the quark masses can be determined in chiral perturbation theory, in particular about the double ratio

\[ Q = \frac{m_s - \hat{m}}{m_u - m_d} = 22.6 \pm 0.8 \]
Summary

- The ratios of the quark masses can be determined in chiral perturbation theory, in particular about the double ratio

\[ Q = \frac{m_s - \hat{m}}{m_u - m_d} = 22.6 \pm 0.8 \]

- Sum rules and accurate $\tau$ data lead to a determination of the strange quark mass. Older values were typically around 125 MeV whereas the most recent ones cluster in the region between 80 and 100 MeV
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- Sum rules and accurate \( \tau \) data lead to a determination of the strange quark mass. Older values were typically around 125 MeV whereas the most recent ones cluster in the region between 80 and 100 MeV.

- Lattice calculations with dynamical fermions and relatively low quark masses have started to appear. Two different groups find values around 80 MeV, whereas a third one around 110 MeV.